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ON A FORMULA FOR $\pi_\nu(x)$

By S. M. Shah

Mathematics Department, Muslim University, Aligarh.

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Let $\pi_\nu(x)$ denote the number of numbers, not exceeding x and which have just ν different prime factors, and $\sigma_\nu(x)$ the number of numbers not exceeding x which have ν prime factors, multiple prime factors being counted multiply.

Landau[1] has shewn that*

$$\pi_\nu(x) \sim \sigma_\nu(x) \sim \frac{1}{(\nu-1)!}\frac{x\,(llx)^{\nu-1}}{\log x}$$

for every fixed value of ν.

I prove here that†

$$\pi_\nu(x) = \frac{1}{(\nu-1)!}\frac{x\,(llx)^{\nu-1}}{\log x} + \frac{B}{(\nu-2)!}\frac{x\,(llx)^{\nu-2}}{\log x} + o\left(\frac{x(llx)^{\nu-2}}{\log x}\right) \quad \ldots \quad (1)$$

We write llx for log log x.

† An empty product (o!) is to be replaced by unity in (1), (2) and throughout the paper.

$$\sigma_\nu(x) = \frac{1}{(\nu-1)!}\frac{x(llx)^{\nu-1}}{\log x} + \frac{B}{(\nu-2)!}\frac{x(llx)^{\nu-2}}{\log x} + o\left(\frac{x(llx)^{\nu-2}}{\log x}\right) \quad \ldots \quad (2)$$

for every fixed value of $\nu \geq 2$.

§1

I have proved[2] that

$$\pi_2(x) = \frac{x\, llx}{\log x} + \frac{Bx}{\log x} + o\left(\frac{x}{\log x}\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where B is a constant.* Hence (1) † is true for $\nu=2$.

Lemma 1.

$$(a) \quad \sum_{p\leq x}\frac{\log^r p}{p} = \frac{1}{r}\log^r x + E_r + O\left(\frac{1}{\log^q x}\right)$$

$(r \geq 1)$. E_r is a constant and q is any fixed integer.

$$(b) \quad \sum_{p\leq x}\frac{1}{p} = llx + B + O\left(\frac{1}{\log^q x}\right)$$

Proof—See *Handbuch*, pp. 201-203.

Lemma 2.

$$\sum_{p<\sqrt{x}}\frac{1}{p(\log x - \log p)} = \frac{llx}{\log x} + \frac{B}{\log x} + \frac{E_1}{\log^2 x} + \ldots + \frac{E_{n-1}}{\log^n x} + O\left(\frac{1}{\log^{n+1} x}\right).$$

where n is any fixed integer.

Proof.

$$\frac{1}{p(\log x - \log p)} = \frac{1}{p\log x} + \frac{1}{\log^2 x}\frac{\log p}{p} + \ldots\ldots + \frac{1}{\log^n x}\frac{\log^{n-1} p}{p} + \frac{\log^n p}{\log^n x}\frac{1}{p(\log x - \log p)}.$$

$$\therefore \sum_{p<\sqrt{x}}\frac{1}{p(\log x - \log p)} = \frac{1}{\log x}\left\{llx - \log 2 + B + O\left(\frac{1}{\log^q x}\right)\right\}$$

* $B = \lim_{x=\infty}\left\{\sum_{p\leq x}\frac{1}{p} - \log\log x\right\}$

† Since $\sigma_2(x) = \pi_2(x) + \pi(\sqrt{x})$ ∴ (2) is also true for $\nu=2$.

$$+\ldots\ldots+\frac{1}{\log^n x}\left\{\frac{\log^{n-1} x}{(n-1)\,2^{n-1}}+E_{n-1}+O\left(\frac{1}{\log^q x}\right)\right\}$$

$$+\sum_{p<\sqrt{x}}\frac{\log^n p}{p\log^n x\,(\log x-\log p)}$$

$$=\frac{1}{\log x}\left\{llx+B-\log 2+\sum_1^{n-1}\frac{1}{k.\,2^k}\right\}+O\left(\frac{1}{\log^{q+1} x}\right)$$

$$+R+\sum_1^{n-1}\frac{E_k}{\log^{k+1} x}$$

where $R=\sum_{p<\sqrt{x}}\frac{\log^n p}{\log^n x\; p\,(\log x-\log p)}$

Consider $S=\sum_{p<\sqrt{x}}\frac{\log^N p}{p\,(\log x-\log p)\log^{N-1} x}$

$$=\sum_2^{[\sqrt{x}]}\frac{I(n)-I(n-1)}{\log n}\;\frac{\log^N n}{n\,(\log x-\log n)\log^{N-1} x}$$

where $I(n)=\sum_{p\leq n}\log p=n+n\,\varepsilon(n)=n+O\left(ne^{-\alpha\sqrt{\log n}}\right)$

where $\alpha>o$.

$$\therefore\; S=\sum_2^{[\sqrt{x}]}\frac{\log^{N-1} n}{n(\log x-\log n)\log^{N-1} x}$$

$$+\sum_2^{[\sqrt{x}]}\frac{n\,\varepsilon(n)-(n-1)\,\varepsilon(n-1)}{\log n}\;\frac{\log^N n}{n\,(\log x-\log n)\log^{N-1} x}$$

$$=\Sigma_1+\Sigma_2.$$

$$\Sigma_1=\sum_2^{[e^{N-1}]}+\sum_{[e^{N-1}]+1}^{[\sqrt{x}]}$$

$$=O\left(\frac{1}{\log^N x}\right)+\int_{[e^{N-1}]+1}^{[\sqrt{x}]}\frac{\log^{N-1} u}{u(\log x-\log u)\log^{N-1} x}\,du$$

$$=O\left(\frac{1}{\log^N x}\right)+I\left([e^{N-1}]+1,\,[\sqrt{x}]\right)$$

$$=O\left(\frac{1}{\log^N x}\right)+I\left(2,\sqrt{x}\right)-I\left(2,[e^{N-1}]+1\right)-I\left([\sqrt{x}],\sqrt{x}\right)$$

$$= O\left(\frac{1}{\log^N x}\right) + I(2, \sqrt{x})$$

since the last two integrals are of $O\left(\frac{1}{\log^N x}\right)$

Now $I(2,\sqrt{x}) = \int_{\log 2}^{\frac{1}{2}\log x} \frac{y^{N-1}\,dy}{\log^{N-1} x.\,(\log x - y)} = \frac{1}{\log^{N-1} x} \quad J_{N-1}$

where J_{N-1} satisfies the relation

$$J_{N-1} = \log x.\,J_{N-2} - \frac{1}{N-1}\left\{\frac{1}{2^{N-1}}(\log x)^{N-1} - (\log 2)^{N-1}\right\}$$

and $J_1 = \log x.\,(\log 2 - \frac{1}{2}) + \log x.\log\left(1 - \frac{\log 2}{\log x}\right) + \log 2.$

$$\therefore J_{N-1} = \log^{N-1} x\left\{\log 2 - 1 + \tfrac{3}{8} - \sum_{3}^{N-1}\frac{1}{k.\,2^k}\right\} + O\left(\frac{1}{\log x}\right)$$

$$\therefore \Sigma_1 = \log 2 - 1 + \tfrac{3}{8} - \sum_{3}^{N-1}\frac{1}{k.\,2^k} + O\left(\frac{1}{\log^N x}\right)$$

Also $\Sigma_2 = \sum_{2}^{[\sqrt{x}]} \frac{n\,\varepsilon(n)\log^{N-1} n}{n(\log x - \log n)\log^{N-1} x}$

$$- \sum_{2}^{[\sqrt{x}]} \frac{n\,\varepsilon(n)\log^{N-1}(n+1)}{(n+1)[\log x - \log(n+1)]\log^{N-1} x}$$

$$- \frac{\varepsilon(1)\log^{N-1} 2}{2(\log x - \log 2)\log^{N-1} x}$$

$$+ \frac{[\sqrt{x}]\,\varepsilon([\sqrt{x}])\log^{N-1}([\sqrt{x}]+1)}{([\sqrt{x}]+1)\{\log x - \log([\sqrt{x}]+1)\}\log^{N-1} x}$$

$$= \sum_{2}^{[\sqrt{x}]} \frac{n\,\varepsilon(n)}{\log^{N-1} x}\left\{\frac{\log^{N-1} x}{n(\log x - \log n)}\right.$$

$$\left. - \frac{\log^{N-1}(n+1)}{(n+1)[\log x - \log(n+1)]}\right\} + O\left(\frac{1}{\log^N x}\right)$$

$$= \Sigma_3 + O\left(\frac{1}{\log^N x}\right)$$

Now

$$\Sigma_3 = \sum_{2}^{[\sqrt{x}]} \frac{n\,\varepsilon(n)}{\log^{N-1} x}$$

$$\left\{\frac{(\log x-\log n)\,\{(n+1)\log^{N-1} n-n\log^{N-1}(n+1)\}+O\,(\log^{N-1} n)}{n\,(n+1)\,(\log x-\log n)\,[\log x-\log(n+1)\,]}\right\}$$

$$=\frac{1}{\log^{N-1} x}\sum_{2}^{[\sqrt{x}]}\frac{n\varepsilon(n)}{n(n+1)}\frac{\{(n+1)\log^{N-1} n-n\log^{N-1}(n+1)\}}{\{\log x-\log(n+1)\}}$$

$$+\sum_{2}^{[\sqrt{x}]} O\left(\frac{n\,\varepsilon(n)}{\log^{N-1} x}\;\frac{\log^{N-1} n}{n^2\,(\log x-\log n)^2}\right)$$

$$=\frac{1}{\log^{N-1} x}\sum_{2}^{[\sqrt{x}]} O\left(\frac{\log^{N-1} n}{\log x-\log n}\;\frac{\varepsilon(n)}{n}\right)+O\left(\frac{1}{\log^{N} x}\right)$$

$$=O\left(\frac{1}{\log^{N} x}\right)$$

$$\therefore\ \Sigma_2 = O\left(\frac{1}{\log^{N} x}\right)$$

$$\text{Hence } S=\Sigma_1+\Sigma_2=\log 2-1+\tfrac{3}{8}-\sum_{3}^{N-1}\frac{1}{k.\,2^k}+O\left(\frac{1}{\log^{N} x}\right)$$

$$\therefore\ R=\frac{1}{\log x}\left\{\log 2-1+\tfrac{3}{8}-\sum_{3}^{n-1}\frac{1}{k.\,2^k}\right\}+O\left(\frac{1}{\log^{n+1} x}\right)$$

$$\therefore\ \sum_{p<\sqrt{x}}{}^{*}\frac{1}{p(\log x-\log p)}=\frac{llx}{\log x}+\frac{B}{\log x}+\sum_{1}^{n-1}\frac{E_k}{\log^{k+1} x}+O\left(\frac{1}{\log^{n+1} x}\right)$$

Lemma 3.

For every fixed $\nu\geq 2$

$$\sum_{p\leq\frac{x}{2}}\frac{1}{p}\frac{\{\log(\log x-\log p)\}^{\nu-1}}{\log x-\log p}$$

$$=\left(1+\frac{1}{\nu}\right)\frac{(llx)^{\nu}}{\log x}+\frac{B\,(llx)^{\nu-1}}{\log x}+o\left(\frac{(llx)^{\nu-1}}{\log x}\right)$$

Proof.

$$\sum_{p\leq\frac{x}{2}}\frac{1}{p}\frac{\{\log(\log x-\log p)\}^{\nu-1}}{\log x-\log p}=\sum_{p<\sqrt{x}}{}_{1}+\sum_{\sqrt{x}\leq p\leq\frac{x}{2}}{}_{2}$$

* q may be chosen $\geq n$.

Where

$$\Sigma_1 = \sum_{p<\sqrt{x}} \frac{\{\log(\log x-\log p)\}^{\nu-1}}{p(\log x-\log p)}$$

$$= \sum_{p<\sqrt{x}} \left\{ (llx)^{\nu-1} + O\left(\frac{(llx)^{\nu-2}\log p}{\log x} \right) \right\} \frac{1}{p(\log x-\log p)}$$

$$= \frac{(llx)^{\nu-1}}{\log x} \left\{ llx + B + o(1) \right\} + O\left(\frac{(llx)^{\nu-2}}{\log^2 x} \sum_{p<\sqrt{x}} \frac{\log p}{p} \right)$$

$$= \frac{(llx)^{\nu-1}}{\log x} (llx + B) + o\left(\frac{(llx)^{\nu-1}}{\log x} \right)$$

Now

$$\Sigma_2 = \sum_{\sqrt{x}\le p\le \frac{x}{2}} \frac{\{\log(\log x-\log p)\}^{\nu-1}}{p(\log x-\log p)}$$

$$= \sum_{[\sqrt{x}]}^{[x/2]} \frac{1}{n\log n} \frac{\{\log(\log x-\log n)\}^{\nu-1}}{(\log x-\log n)}$$

$$+ \sum_{[\sqrt{x}]}^{[x/2]} \frac{n\,\varepsilon(n)-(n-1)\,\varepsilon(n-1)}{\log n} \left\{ \frac{\{\log(\log x-\log n)\}^{\nu-1}}{n(\log x-\log n)} \right\}$$

$$= \Sigma_3 + \Sigma_4$$

where $\Sigma_3 = \displaystyle\sum_{[\sqrt{x}]}^{[x/2]} \frac{\{\log(\log x-\log n)\}^{\nu-1}}{n\log n(\log x-\log n)}$

$$= \int_{\sqrt{x}}^{x/2} \frac{\{\log(\log x-\log u)\}^{\nu-1}}{u\log u(\log x-\log u)}\,du + O\left(\frac{1}{\log x}\right)$$

by applying Euler Maclaurin sum formulæ.

Further $\Sigma_4 = \displaystyle\sum_{[\sqrt{x}]}^{[x/2]} \frac{n\,\varepsilon(n)-(n-1)\,\varepsilon(n-1)}{\log n} \left\{ \frac{\{\log(\log x-\log n)\}^{\nu-1}}{n(\log x-\log n)} \right\}$

$$= \sum_{[\sqrt{x}]}^{[x/2]-1} n\,\varepsilon(n) \left[\frac{\{\log(\log x-\log n)\}^{\nu-1}}{n\log n(\log x-\log n)} \right.$$

$$\left. - \frac{\{\log[\log x-\log(n+1)]\}^{\nu-1}}{(n+1)\log(n+1)[\log x-\log(n+1)]} \right]$$

$$+ [x/2]\,\varepsilon([x/2]) \left[\frac{1}{[x/2]} \frac{\{\log(\log x-\log[x/2])\}^{\nu-1}}{\log x-\log[x/2]} \frac{1}{\log[x/2]} \right]$$

$$- \frac{\{[\sqrt{x}]-1\}\,\varepsilon([\sqrt{x}]-1)}{[\sqrt{x}]\log([\sqrt{x}])} \quad \frac{\{\log(\log x-\log[\sqrt{x}])\}^{\nu-1}}{\log x-\log[\sqrt{x}]}$$

$$= \Sigma_5 + \Sigma_6 - \Sigma_7$$

Then $$\Sigma_6 = O\left(\frac{1}{\log x}\right)$$

$$\Sigma_7 = O\left(\frac{1}{\log x}\right)$$

$$|\Sigma_5| < K \sum_{[\sqrt{x}]}^{[x/2]-1} \frac{n}{\log^q n} \left| \frac{\left\{\log\log\frac{x}{n}\right\}^{\nu-1}}{n\log n\log\left(\frac{x}{n}\right)} - \frac{\left\{\log\log\frac{x}{n+1}\right\}^{\nu-1}}{(n+1)\log(n+1)\log\left(\frac{x}{n+1}\right)} \right|$$

$$= O\left((llx)^{\nu-1}\sum_{[\sqrt{x}]}^{[x/2]-1}\frac{1}{n\log^q n}\right) = O\left(\frac{1}{\log x}\right)$$

$$\Sigma_2 = \int_{\sqrt{x}}^{x/2} \frac{\{\log(\log x-\log u)\}^{\nu-1}}{u\log u(\log x-\log u)}\,du + O\left(\frac{1}{\log x}\right)$$

$$= \int_{\log 2}^{\frac{\log x}{2}} \frac{dy\,(\log y)^{\nu-1}}{y(\log x-y)} + O\left(\frac{1}{\log x}\right)$$

$$= \frac{1}{\log x}\left[\int_{\log 2}^{\frac{\log x}{2}} \frac{(\log y)^{\nu-1}}{y}\,dy + \int_{\log 2}^{\frac{\log x}{2}} \frac{(\log y)^{\nu-1}}{\log x-y}\,dy\right] + O\left(\frac{1}{\log x}\right)$$

$$= \frac{I_1+I_2+O(1)}{\log x}$$

where $$I_1 = \int_{\log 2}^{\frac{\log x}{2}} \frac{(\log y)^{\nu-1}}{y}\,dy = \int_{ll2}^{\log\left(\frac{\log x}{2}\right)} v^{\nu-1}\,dv$$

$$= \frac{1}{\nu}(llx)^{\nu} - \log 2.\,(llx)^{\nu-1} + o\left((llx)^{\nu-1}\right)$$

$$I_2 = \int_{\log 2}^{\frac{\log x}{2}} \frac{(\log y)^{\nu-1}}{\log x-y}\,dy = \int_{\frac{\log x}{2}}^{\log x-\log 2} \frac{\{\log(\log x-y)\}^{\nu-1}}{y}\,dy$$

$$= \int_{\frac{\log x}{2}}^{\log x-\log 2} \frac{dy}{y}\left\{(llx)^{\nu-1} + O\left((llx)^{\nu-2}\frac{y}{\log x}\right)\right\}$$

$$= (llx)^{\nu-1}\{\log(\log x-\log 2)-\log(\tfrac{1}{2}\log x)\} + O\left((llx)^{\nu-2}\right)$$

$$= \log 2\ (llx)^{\nu-1} + o\left((llx)^{\nu-1} \right)$$

$$\therefore\ \Sigma_2{}^* = \frac{1}{\log x}\left\{\frac{1}{\nu}\ (llx)^{\nu} + o\left((llx)^{\nu-1} \right)\right\} + O\left(\frac{1}{\log x}\right) \quad . \quad . \quad \text{(B)}$$

$$\therefore\ \Sigma_1 + \Sigma_2 = \left(1+\frac{1}{\nu}\right)\frac{(llx)^{\nu}}{\log x} + \frac{B\,(llx)^{\nu-1}}{\log x} + o\left(\frac{(llx)^{\nu-1}}{\log x}\right)$$

Lemma 4.

$$\sum_{p\leq\frac{x}{2}} \frac{1}{p\,(\log x - \log p)} = \frac{2\,llx}{\log x} + O\left(\frac{1}{\log x}\right)$$

This follows from Lemma 2 and (B).

Assume now (1) to be true for $\nu = n \geq 2$. We prove it to be true for $\nu = n+1$.

We have

$$\pi_{n+1}(x) = \frac{1}{(n+1)} \sum_{p\leq\frac{x}{2}} \pi_n\left(\frac{x}{p}\right) + o\left(\frac{x\,(ll\,x)^{n-1}}{\log x}\right)^{1}$$

Further

$$\sum_{p\leq x/2} \pi_n\left(\frac{x}{p}\right) = \frac{x}{(n-1)!}\sum_{p\leq x/2}\frac{1}{p}\,\frac{\{\log(\log x - \log p)\}^{n-1}}{\log x - \log p}$$

$$+ \frac{Bx}{(n-2)!}\sum_{p\leq x/2}\frac{1}{p}\,\frac{\{\log(\log x - \log p)\}^{n-2}}{\log x - \log p}$$

$$+ \ x \sum_{p\leq x/2} \alpha\left(\frac{x}{p}\right)\frac{1}{p}\,\frac{\{\log(\log x - \log p)\}^{n-2}}{\log x - \log p}$$

$$= \frac{x}{(n-1)!}\,\Sigma_1 + \frac{Bx}{(n-2)!}\,\Sigma_2 + x\,\Sigma_3 \text{ (say)}$$

where $\left|\alpha\left(\frac{x}{p}\right)\right| < \varepsilon$ if $\frac{x}{p} \geq x_0 = x_0(\varepsilon) \geq 3$

$$\text{Now } \Sigma_3 = \sum_{p\leq\frac{x}{x_0}}{}_{4} + \sum_{\frac{x}{x_0}<p\leq\frac{x}{2}}{}_{5}$$

$$|\Sigma_4| < \varepsilon \sum_{p\leq\frac{x}{x_0}} \frac{1}{p}\,\frac{\left\{\log\log\left(\frac{x}{p}\right)\right\}^{n-2}}{\log x - \log p}$$

*This result is true when $\nu = 1$.

$$< K_1 \varepsilon \frac{(llx)^{n-1}}{\log x}, \quad \text{by lemma 3 if } n \geqq 3 \text{ and by lemma 4 if } n = 2.$$

$$|\Sigma_5| = \left| \sum_{\frac{x}{x_0} < p \leq \frac{x}{2}} \alpha\left(\frac{x}{p}\right) \frac{1}{p} \frac{\left\{\log\log\left(\frac{x}{p}\right)\right\}^{n-2}}{\log x - \log p} \right|$$

$$< K_2 \sum_{\frac{x}{x_0} < p \leq \frac{x}{2}} \frac{1}{p} \frac{\left\{\log\log\left(\frac{x}{p}\right)\right\}^{n-2}}{\log x - \log p}$$

$$< \frac{K_2}{\log 2} (llx_0)^{n-2} \sum_{\frac{x}{x_0} < p \leq \frac{x}{2}} \frac{1}{p} < K_3 \frac{(llx_0)^{n-1}}{\log x}$$

$$\therefore \Sigma_3 = \Sigma_4 + \Sigma_5 = o\left(\frac{(llx)^{n-1}}{\log x}\right)$$

$$\therefore \sum_{p \leq \frac{x}{2}} \pi_n\left(\frac{x}{p}\right) = \frac{x}{(n-1)!}\left\{\left(1+\frac{1}{n}\right)\frac{(llx)^n}{\log x} + B\frac{(llx)^{n-1}}{\log x} + o\left(\frac{(llx)^{n-1}}{\log x}\right)\right\}$$

$$+ \frac{Bx}{(n-2)!}\left\{\frac{n}{n-1}\frac{(llx)^{n-1}}{\log x} + O\left(\frac{(llx)^{n-2}}{\log x}\right)\right\}$$

$$+ o\left(\frac{x(llx)^{n-1}}{\log x}\right)$$

$$= (n+1)\left[\frac{1}{n!}\frac{x(llx)^n}{\log x} + \frac{B}{(n-1)!}\frac{x(llx)^{n-1}}{\log x} + o\left(\frac{x(llx)^{n-1}}{\log x}\right)\right]$$

$$\therefore \pi_{n+1}(x) = \frac{1}{n!}\frac{x(llx)^n}{\log x} + \frac{B}{(n-1)!}\frac{x(llx)^{n-1}}{\log x} + o\left(\frac{x(llx)^{n-1}}{\log x}\right)$$

which proves (1) for $\nu = n + 1$. Hence (1) is proved.

To prove (2) we note that, if $n \geqq 3$

$$0 \leq \sigma_n(x) - \pi_n(x)$$
$$\leq \rho_1(x) + \ldots + \rho_{n-2}(x) + A_{n-1}(x)$$

where ρ_ν (x) denotes the number of integers, not exceeding x which have ν different prime factors,

and

$$A_{n-1}(x) = \sum_{p_1^2 p_2 \ldots p_{n-1} \leq x} 1 \quad \left(p_1 \gtrless p_2 \gtrless p_3 \quad \gtrless p_{n-1}\right)$$

$$= O\left[\sum_{p\leq\sqrt{x}} \pi_{n-2}\left(\frac{x}{p^2}\right)\right]$$

$$= O\left[\sum_{p\leq\sqrt{\frac{x}{2}}} \pi_{n-2}\left(\frac{x}{p^2}\right)\right]$$

$$= O\left[\sum_{p\leq\sqrt{\frac{x}{2}}} \frac{x}{p^2}\,\frac{\left(\log\log\frac{x}{p^2}\right)^{n-3}}{\log x - 2\log p}\right]$$

But
$$\sum_{p\leq\sqrt{\frac{x}{2}}} \frac{1}{p^2}\,\frac{\left(\log\log\frac{x}{p^2}\right)^{n-3}}{\log x - 2\log p} = \sum_{p\leq x^{\frac{1}{4}}} + \sum_{x^{\frac{1}{4}}<p\leq\sqrt{\frac{x}{2}}}$$

$$= o\left(\frac{(llx)^{n-2}}{\log x}\right) + O\left(\frac{(llx)^{n-3}}{x^{\frac{1}{8}}}\right)$$

$$\therefore\ A_{n-1}(x) = o\left(\frac{(llx)^{n-2}}{\log x}\right)$$

Further
$$\rho_\nu(x) \sim \frac{1}{(\nu-1)!}\,\frac{x(llx)^{\nu-1}}{\log x}$$

$$\therefore\ 0 \leq \sigma_n(x) - \pi_n(x) = o\left(\frac{x(llx)^{n-2}}{\log x}\right) \qquad \ldots \qquad \ldots \quad (4)$$

From (1) and (4) we get (2) if $\nu\geq 3$. We have already seen that (2) is true if $\nu=2$. Hence (2) is proved for $\nu\geq 2$.

References

1. Landau, *Handbuch*, pp. 205—213.
2. Shah, *Bombay University J.*, Vol. 2, Part II, pp. 35—39.

THE MATHEMATICAL THEORY OF A NEW RELATIVITY

By SHAH MUHAMMAD SULAIMAN,

CHIEF JUSTICE, HIGH COURT, ALLAHABAD

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INTRODUCTION

In Chapter III* the existing theories about the expansion of the Universe have been, for economy of space, very briefly criticised, and it is submitted that extraordinary assumptions are made in these and that they are also wholly inadequate. The impossibility of a cosmical force of repulsion, acting at a distance, and increasing with the distance between two bodies, has been pointed out. It has also been shown that the Rotational Theory of Light, published in 1933 to reconcile the phenomena of interference, diffraction and scintillation, can partially explain the recessional velocities of nebulæ. This assumption is not essential for the main theory, but its purpose is merely to show that the apparent recessional velocities are in part spurious, and that the huge mass of a distant nebula like the one in Ursa Majoris is not running away from us at anything like 30,000 kms. per second.

In Chapter IV a natural assumption has been made that emanations from particles of matter are not confined to our present range of observation, but that corpuscles called gravitons, finer than light corpuscles, are also emitted, though they are beyond our vision. But even this assumption is not essential for the theory. As shown in Section V, one may start simply with the known fact that light radiations, *i.e.*, swarms of light corpuscles or radions, emanate from every part of an outer shell of a luminous body. The assumption made by Einstein and de Sitter that the velocity of light in space is constant is accepted as a first approximation. But their further assumption that the velocity of light relative to two moving bodies is always the same, no matter with what

*The first two chapters of this paper were printed in the Proceedings of the Academy of Sciences, U.P., India, Vol. 4, Part 1, pp. 1—36 (August, 1934) and dealt with the law of gravitation.

different velocities they may be moving, has been rejected; and the obvious assumption is made that the velocity of light relative to the front part of a moving body is different from that relative to its rear when light moves in opposite directions. From this simple fact the acceleration of a moving body which is emanating matter is deduced dynamically, furnishing a correction to Newtonian mechanics. The cosmological principle in Relativity that the picture of the Universe is exactly the same no matter from what point we observe it has been rejected. The apparent effect that all distant nebulæ seem to be moving away from the earth is explained on the simple principle of components of accelerations and velocities. The Universe need no longer be scattering away, due to any supposed explosion from a condensed spherical shell which suddenly burst ages ago.

In Chapter V the assumption in Special Relativity that light from one moving body to another moving body takes the same time as light from the second body to the first, no matter how different their velocities may be, as well as the definition of common time between two bodies determined by a single journey of light are not accepted. Common time is defined as the whole time taken by light from one body to the other in performing the double journey. It is shown dynamically that both bodies, employing such a messenger, measure the common time as well as the relative distance between them exactly alike. The velocity of light is shown not to be absolute; but only the average velocity of light in a to-and-fro journey is nearly absolute. In view of the admitted fact that "All experimental methods of measuring the velocity of light determine only an average to-and-fro velocity" (Eddington[1]), it is wholly unnecessary to assume the absoluteness of a single journey velocity of light, and it is quite sufficient to accept that in a double journey the velocity is nearly absolute; and so no terrestrial experiment detects the motion of the earth, the error being of too small an order.

Newton's Mechanics was designed for an omniscient superman, who could see distant objects instantaneously, and measure time and space also instantaneously, which meant an employment of a messenger travelling with infinite velocity, so that simultaneous velocities and accelerations of several bodies could be ascertained at once. To Newton forces moved with infinite velocities and acted instantaneously, and so it made no difference whether the influenced body was stationary or was moving with a finite velocity. But a mere human being can employ only light as his fastest messenger, which travels with only finite velocity. And the forces which he experiences neither travel with infinite velocity

nor act instantaneously. Newton's Mechanics therefore requires a correction when applied to bodies moving with high velocities. Einstein gets over the difficulty by attributing to the velocity of light some of the properties of infinity, *e.g.*, of absoluteness and maximum speed, even though these necessitate contraction of length, extension of time and increase of mass with velocity, and involve a curvature of space, making the Universe finite and surprisingly small, and the space itself expanding into non-space.

As a substitute for Einstein's Relativity theory, only one single assumption, which is natural and should be obvious, is made that all influences take time to act and do not act instantaneously, which in dynamical language means that they are propagated with finite and not infinite velocities.

All terrestrial experiments involving the addition of velocities, like those of Hoek, Fresnel and Fizeau, are explained by the *first* necessary deduction that the relative velocity between two bodies moving with velocities v' and u, measured by employing a messenger travelling with velocity D in a to-and-fro journey, is different when both the bodies are moving than when one is reduced to rest, and is given by the formula

$$\frac{v}{v'-u} = \frac{1+\frac{v'}{D}-\frac{u}{D}}{\left(1+\frac{v'}{D}\right)\left(1-\frac{u}{D}\right)} = \frac{1}{1-\frac{v'u}{D^2}} \text{ nearly.}$$

All astronomical observations, including the advance of the perihelia of planets, the deflection of light and the shift of the spectral lines (already explained in Chapters I and II) and all terrestrial experiments, like those of Michelson and Morley, Trouton and Nóble, Kaufmann and Bucherer, are explained by the *second* necessary deduction that wherever there is a field of force which takes time to act and so can be represented by a velocity of flow D, no matter whether it be of gravitation or radiation, or electric or magnetic field, then its action on a body moving with velocity v inclined at an angle θ to the direction of the flow, is exactly the same as if the body were stationary and the direction of the flow of the same magnitude were shifted forward by an angle α given approximately by the ratio applicable to the compounding of the two velocities $D \sin\alpha = v \sin\theta$. So that when the angle is a right angle the effective force is decreased by the factor $\cos\left(\sin^{-1}\frac{v}{D}\right) = \sqrt{1-\frac{v^2}{D^2}}$.

All the problems like Sommerfeld's fine structure of spectral lines are explained by the *third* necessary deduction that when a force travelling with a finite velocity D acts on a thin spherical shell of radius a,

spinning with an angular velocity w round an axis through its centre perpendicular to the force, the effect is to decrease or increase its spin according as the force is repulsive or attractive, but always to decrease the component of the force along the diameter by the factor $\sqrt{1-\frac{a^2w^2}{D^2}}$ nearly.

With the help of these generalised laws, Galileo's and Newton's Mechanics, with proper corrections for moving bodies, is completely restored; and although Einstein's Theory of Relativity is not accepted, all its practical results are deduced in full.

I must again express my gratitude to Dr. D. S. Kothari, M.Sc., Ph.D., and Mr. A. N. Chatterji, M.Sc., for their very kind help, similar to that given before.

CHAPTER III

Anomalies in the Existing Hypotheses

SECTION I

THE EXPANDING UNIVERSE

1. *The Observed Facts.*—Slipher discovered in 1912 (confirmed in 1922) that the spiral nebulæ possess radial velocities, predominantly of recession, and large as compared with the ordinary range of stellar velocities. In 1929, Dr. Hubble found that the velocity of recession was greater the greater the distance, and enunciated the law that the mean velocity of recession at a given distance is proportional to the distance. The observations show clearly that the radial velocities appear to obey a linear law of increase, the velocities being simply proportional to the distances. The individual recessional velocities do not obey the law accurately, but the deviations in radial velocity from the velocity distance proportionality are relatively small, perhaps of the order of 80 km. per sec. Another extraordinary observed fact is that all the galaxies seem almost without exception to be running away fast from our solar system. One nebula, forming a faint cluster in the constellation Germini is found to be receding at a speed of 24,000 kms. per second, *i.e.*, about the speed of an alpha particle. Its distance is about 150 million light years. It is now reported that a nebula in Ursa Majoris is moving away with a velocity equal to about one-tenth of the velocity of light. Observations show that outside a sphere of a little over a million light years radius round our galaxy, 80 nebulæ are observed to be moving outwards, and not one has been observed moving inwards. Out of 90 galaxies

which have been observed, only five are found to be moving towards our system; but even these five exceptions are confined to the very nearest of the nebulæ and their velocities of approach are not large. Even after corrections for the motion of the Sun in its orbit round the centre of the galaxy have been made, the velocities of approach of the nearest nebulæ are considerably reduced, but still they are not found to possess velocities of recession proportional to their distances. It is, however, believed that allowing for all possibilities of error and misinterpretation, there is a residual expansion, and the super-system of galaxies is dispersing as if it were a puff of smoke or as if a gas suddenly released would expand.

2. *The Inference.*—As there is absolutely no reason why all the nebulæ should be running away from the earth as the centre of expansion, the inference is a law of general uniform expansion, that is each individual body is receding from every other body at a rate proportional to the distance between them. The serious difficulty is that for a finite space this implies an explosion from a centre, and the particles must scatter away with velocities increasing at a rapidly progressive rate.

SECTION II

THE INADEQUACY OF CLASSICAL THEORIES

1. *Newton's Corpuscular Theory.*—If light be regarded as a swarm of bullets travelling in straight lines through space, as Newton imagined, there would seem to be no reason why they should lose any part of their velocities or energy on the way, if Newton's first law of motion holds good. The only obstruction they can meet would be from material particles, which will either absorb them or scatter them, and not cause any change in frequency.

2. *Huygens' Wave Theory.*—The wave theory of light also cannot explain how the ether waves can lose their energy partially in their passage through space. Waves do not obstruct each other, but pass through. The law of the conservation of energy is contrary to any such automatic loss.

3. *Plancks' Quantum Theory.*—The observed shifting of the spectrum of a nebula towards the red signifies lower frequency of light waves, which, according to the Quantum Theory, would mean lower energy, so that if for any cause a light quantum can lose some of its energy in travelling through space, the reddening would be accounted for, and the loss of energy would be proportional to the distance. But neither the wave theory nor the corpuscular nor the quantum theory has succeeded in explaining how such a loss can occur. As pointed out by

Eddington[2] "there is nothing in the existing theories of light (Wave Theory or Quantum Theory) which justifies the assumption of such a loss, (*i.e.*, loss of energy proportional to distance)".

4. *Zwicky's*[3] *Theory.*—Zwicky suggested that light by its gravitational effects parts with its energy to the material particles thinly strewn in intergalactic space which it passes on its way. But the numerical result obtained after allowing for such gravitational effects showed that the theory was fallacious. The loss of energy due to gravitational effects would not be sufficient to account for the extent of the reddening observed. He has also suggested that the gravitational pull of stars and nebulæ on light passing near them causes reddening, just as bending of starlight at an eclipse of the sun. But no adequate explanation has been given as to how the pull of stars can redden the light; it will merely deflect it.

5. *Macmillan's Suggestion.*—Macmillan[4] suggested that the red shift may be due to the loss of energy in the photon in course of time, due either to inherent instability or collisions with other photons. One can heartily agree with his conclusion that "Such an interpretation of the extraordinary shifts that are observed will be more acceptable to many than an interpretation which makes our galaxy a centre from which all others are fleeing with speeds that are proportional to the distances." But unfortunately there is no explanation of any inherent instability in photons; and the straight collisions would scatter them away.

SECTION III

RELATIVITY THEORY

1. *Einstein's Universe.*—Einstein abolished infinity and modified his original equation so as to make space at infinite distances bend round itself and close up. The new equation $G_{\mu\nu} = \lambda g_{\mu\nu}$ contains a cosmical constant. Two forces—the Newtonian attraction and the cosmical repulsion—oppose each other and create an Einstein's universe in unstable equilibrium. In order that a particle placed anywhere may remain at rest, Einstein's world has to be filled with rather too much matter having negligible velocity, and spread over uniformly through space. "This is like restoring a crudely material aether regulated however by the strict injunction that it must on no account perform any useful function, lest it upset the principle of relativity."[5] The

supposition is contrary to observation, as the universe we see has considerable matter moving with great velocity.

2. *De Sitter's Universe.*—De Sitter assumed the density of matter to be infinitely small so as to make the Newtonian attraction negligible, and cause the cosmical repulsion to produce an expansion. But in point of fact, even de Sitter's world requires a large quantity of matter forming a sort of mass-horizon. "He has merely swept the dust away into unobserved corners."[6] "Einstein's universe contains matter, but no motion, and de Sitter's contains motion, but no matter." It is clear that the actual universe containing both matter and motion does not correspond exactly to either of these abstract models. When observed from inside, an empty or motionless universe simply does not exist.

3. *Lemaitre's Solution.*—Lemaitre's theory is that the world began with a violent projection from the state in which it was condensed to a point or atom; and it has now got past the Einstein state. The supposition is that the density of the universe does not depend on position, but is uniform in space, though varies with time. The gravitational effect is taken as nil. The hypothesis involves the assumption of a neutral zone, materialised by a thin spherical shell of negligible mass and no interior stresses, separating altogether the matter inside the condensation from the matter outside, so that matter crossing the shell from outside or from inside is forced to rebound at its boundary. All particles outside the shell are endowed with a hyperbolic radial velocity of escape, while all the matter inside the shell has an elliptic radial velocity and so must fall back on the centre. It is hardly necessary to point out that such a bifurcation is extremely arbitrary.

4. *Eddington's Hypothesis.*—Eddington[7] supposes that primordial material might have consisted of hydrogen (or free protons or electrons). The formation of condensation somehow had the start of the conversion of mass into radiation; thus the effect of conversion of radiation into material mass was to make the universe expand. If primordial material consisted of only matter and no radiation, then the conversion of mass into radiation would occur first, resulting in a contraction. In any case, Eddington's hypothesis is unable in itself to explain an expansion proportional to linear distance.

It has been suggested by A. C. Banerji[8] of Allahabad that the breaking-up of a photon into an electron and a positron, known as the process of ***electrofission***, may explain why the Universe started expanding from the Einstein Universe. Starting with the relativity assumption that "mass for mass matter exerts less gravitational attraction than radiation," he

inferred that the conversion of radiation into matter will lesson the gravitational factor and so if by some method the photon breaks up into two material particles, the Einstein Universe will start expanding.

5. The statement that mass for mass matter exerts less gravitational attraction than radiation is only true even in Relativity when the influence of matter on light passing transversely is concerned. Light itself is not known to exert any gravitational attraction on matter when passing by it. And light is also not at all known to have any gravitational interaction with itself. So if photons only existed, there would have been no gravitation; and if a photon can at all split up into two material particles they would bring into existence a gravitational force for the first time, which would counteract any expansive tendency. Electrofission could not therefore stimulate expansion, particularly as the condensation of radiation into matter would diminish pressure by reducing velocities. In Relativity "the effect of the gravitation of the sun on a light wave, or a very fast particle, proceeding radially is actually a *repulsion.*"[9] Thus if for some reason condensations began to form, light waves would be repelled from such centres of condensation, resulting in some expansion. But this can be so only in a four-dimensional universe, while the motions of the nebulæ which we actually observe are strictly three-dimensional.

SECTION IV

MILNE'S WORLD STRUCTURE

1. Milne[10] adopts Einstein's Special Theory of Relativity in its kinematical aspects, and assumes that two observers in uniform relative motion have identical views of the universe. The world is supposed to have originated with a swarm of particles moving in straight lines, each with a uniform velocity without collisions, possessing an entirely arbitrary velocity distribution. After lapse of sufficient time, the outward moving particles will move into the empty space outside an imaginary sphere, the faster will gain on the slower and the fastest moving will form an expanding spherical frontier zone followed by the next fastest. The inward moving particles will move inward, traverse a chord of the sphere, emerge at the other side and then move outwards.

(1) The theory ignores the effect of gravitational interaction altogether. (2) It evades the possibility of collisions. (3) It assumes that the position is identical, whether the point of observation is near the surface or near the centre. (4) It also assumes that sufficient time has

already elapsed for things to adjust themselves—a purely arbitrary assumption, as there is no reason why some slow particles should still not be moving from distant regions towards the centre, particularly if space is not finite. (5) To provide distribution of nebulæ up to 150 million light years, high speeds must be more frequent than low speeds- an anti-Maxvillion distribution of speeds in a compact aggregation of galaxies.[11] (6) It starts with a swarm of particles moving in all directions without giving any explanation how they came to be so moving at the start. (7) There is difficulty in supposing that the velocities would always be proportional to distance from the centre. Two nebulæ at different distances moving with nearly equal velocity towards the centre can never follow the proportionality of distance law measured from the centre, even though they have passed beyond it. (8) The mathematical equations yield the result $dr/dt = r/t$, which shows that the velocities vary with time and are different at different epochs. This equation is not the same thing as $dr/dt = kr$, where k is a constant in time as well as in space. (9) Milne's universe behaves as if the system was created at the natural origin of time t equal to zero, and the expansion is inevitable and an essential feature of evolution.

2. In addition to the cosmological principle that all particles in the universe are equivalent in their relationship, the hydrodynamical equations of motion and continuity are employed.[12] The universe is treated as if consisting of a huge mass of fluid with all the properties of a fluid including continuity and excluding the existence of singularities. At every point, the motion is one of flow, without acceleration. In contravention with the ordinary ideas of Newtonian gravitation, the nebulæ can only be in uniform rectilinear motion, whatever their interaction, *i.e.*, whatever the law of gravitation. Milne's solution is only the limiting case of the general relativity treatment of Einstein's field equations representing expanding universes, when the gravitational interaction of the particles tends to zero of the general relativity solution.[13]

As the universe we see consists of vast condensations of matter and huge void spaces in between, hydrodynamical principles can hardly with any justification be applied to obtain a formula for expansion of any region when seen from inside, nor can the gravitational effect be properly ignored.

Of course, if the visible region of the Universe, including our Galactic system and the observed nebulæ, even when seen from inside, can be wrongly assumed to constitute a continuous fluid, then no elaborate mathematics is required for the conclusion that a general expansion must

obey the proportionality to distance law. If when any fluid is heated, every region of it expands and pushes out its neighbouring regions, then the whole expansion must follow that law, but not so if the heating is non-uniform.

SECTION V

THE IMPOSSIBILITY OF COSMICAL REPULSION

1. Apart from the extraordinary assumptions as to curvature of space, incomprehensible properties of the velocity of light, variability of time and mass with velocity and numerous other unconvincing results, Relativity leads inevitably to a cosmic force of repulsion, in addition to the apparent force of attraction. This inexplicable "cosmical repulsion" not only acts at a distance, but unintelligibly increases in intensity with the distance between two bodies. Such a concept is philosophically an impossible one. If one body were to influence another body, it is logical to assume that the influence would be greater when the distance between them is smaller. It is altogether illogical to suppose that their influence on each other is greater when the distance is greater.

2. The hypotheses necessarily involves a force of repulsion directly proportional to the distance from every-where. It is like a dispersing force so that the whole system is scattering away, each particle increasing its velocity rapidly. As Eddington puts it "repulsion has no centre, *i.e.*, every point is a centre of repulsion."

Einstein was himself doubtful as to the feasibility of introducing a cosmical repulsion, and considered it "theoretically unsatisfying." But it has now become an essential element not only in de Sitter's theory, but also in the theory propounded by Weyl.

3. If there were a general expansion of the universe then there is no reason why the stars should not recede from all the other stars at the same rate, as explosion would be effective in all directions. We would then see not only stars receding from the earth, but also separating from each other, and thereby increasing the subtended angle on the earth. But in point of fact there is no such high transverse velocity observed between two stars. As every part of space is supposed to be expanding the inflation ought to affect distances between planets in the solar system, electrons in an atom, that is to say, atoms, human beings and every thing on the earth should expand at the same rate as the universe. The expansion might be small, but should exist, though in these cases they may not be observable.

But Eddington suggests that the inflation would be uniform only if the density is uniform, and concludes that only the intergalactic

distances expand, the galaxies themselves are unaffected and all lesser systems like star clusters, stars, human observers and their apparatus and atoms are entirely free from expansion.

4. As in Relativity all change is relative, the expansion of the universe can with equal proportion be explained by a supposed shrinking of our material standards. As Eddington[14] has put it "The theory of the expanding universe might also be called the theory of the shrinking atom. We suppose that we are always the same and that our environment changes ; but we ourselves may be shrinking."

5. The rule that the receding velocity of a spiral nebula is proportionate to its distance is not obeyed by every individual nebula. Furthermore the observed velocities of the five nearest nebulæ do not follow the linear law of increase strictly. They have high velocities of approach, which can be reduced if the rotation of the galactic system is taken into account. But no amount of corrections can convert their velocities to those of recession proportional to their distances. These nebulæ are exceptions to the supposed law of expansion, and even a solitary exception would destroy the law.

6. When the Universe is observed from inside, *i.e.*, from the earth it is found to consist of large condensations of mass moving relatively to each other, and with vast empty spaces in between. When observed from inside, it is, therefore, impossible to accept Einstein's assumption that it is motionless, or de Sitter's assumption that it is empty, or Milne's supposition that it is continuous like a fluid.

7. Relativity attempts to explain the expansion by the suppositions that space is curved and bends round itself, that the nebulæ are all situated on an imaginary three dimensional skin of a four dimensional continuum, and that this unimaginable hollow sphere is expanding by an apparent cosmical force of repulsion increasing with the distance. If the interaction of a hundred thousand million galaxies (nebulæ) be also taken into account, then Relativity would require a space of four hundred thousand million minus four dimensions to explain the Universe!

SECTION VI

THE ROTATIONAL THEORY OF LIGHT

Although the classical and modern theories fail to explain how energy can be lost merely by passage through space, the Rotational Theory of Light put forward in Part II of my Unified Theory of Physical Phenomena (1933) can *partly* account for it.

1. First, if light consists of a swarm of particles, rotating with a period round the path of their forward motion, then while travelling forward their side collisions with other particles in space may, without substantially affecting their forward velocity, reduce their rotational velocity and so alter their frequency. If space is full of fine particles the frequency of light would be reduced in proportion to the distance travelled. The light would become proportionately redder. The more distant a nebula is the longer has light to travel from it, and the more it loses its rotational motion, and the more it becomes red. The loss of energy proportional to distance itself suggests that the change is in some way the result of passage through space. The apparent shift of light to the red side of the spectrum can be partly explained by the progressive loss of rotational motion or angular momentum of the radions. The forward velocity remains unaffected, because either radions collide and are scattered, or they pass through unobstructed, with diminished angular motion. The slight deflections nullify themselves on the average.

Compton has discovered that radiation is both deflected and reddened when it encounters electrons in space. In his experiment when X-rays encountered electrons, the spectral line suffered a shift towards the red, *i.e.*, the wave length increased. "The impact reduces the rotational velocity, it increases the period and therefore λ the wave length, which is the longitudinal motion during the increased time, also increases. And so radiation gets reddened . . . a good part of the observed recession of the stars can be accounted for by the reddening of light as it travels through space."[15]

2. Secondly, observations may perhaps also confirm that the stars in such parts of the galactic system as contain more matter are redder than even more distant stars. Light from a number of globular clusters, at about equal distances from us, but so selected that the amount of intervening gravitational matter varied greatly was examined by P. ten Bruggencate and found to vary according to the intervening matter.[16] The pull of the stars and the nebulæ on passing close to them would combine to increase the radius of the rotational motion, thus decreasing the frequency, but would cancel each other so far as the forward velocity is concerned. In this way also a good part of the reddening can be explained.

3. Thirdly, there can be an inherent loss of angular momentum by the radions in a beam of light colliding against each other while rotating. The loss of angular momentum would mean a decrease in angular velocity which in rotation would correspond to decrease in frequency.

Thus loss in frequency would be proportional to the whole time taken that is proportional to the distance travelled.

4. In this way a very large part of the spectral shift may be the result of the rotation of the radions, and therefore, a greater part of the velocities of recession may be spurious.

The loss in frequency due to these causes would be

$$-\triangle\nu = \nu - (\nu + \triangle\nu) = g.r,$$

where r is the distance travelled by light and g is a constant. From this we get

$$\frac{c}{\lambda} - \frac{c}{\lambda + \triangle\lambda} = gr \quad \therefore \triangle\lambda = \frac{g}{c}\frac{\lambda^2}{1 - \frac{gr}{c}\lambda}.\ r = \frac{g\lambda^2}{c}.\ r \text{ nearly} \quad \ldots \quad (18\ 1)$$

5. Whether light passing through intensely cold regions of space also automatically loses some energy can be tested by letting two beams of monochromatic light from one source pass through two separate and long vacuum chambers and then interfere, and observing the changes, if any, in the interference fringes when one chamber is heated and the other cooled.

For a detailed consideration of the Rotational Theory of Light, reference may be made to the Unified Theory of Physical Phenomena, Part II (1933).

CHAPTER IV

The Emission Theory of Matter

SECTION I

THE EMISSION THEORY OF GRAVITATION

In my Unified Theory of Physical Phenomena, published in 1933, I pointed out that action at a distance was illogical, and that the apparent force of attraction was nothing but motion caused by an internal action in matter. It was pointed out at some length that Newton's Law of Gravitation between two stationary bodies could be very easily deduced from the assumption that every particle of matter is emitting in all directions small corpuscles called "gravitons" and that the rate of emission depends on the material potential in its immediate neighbourhood, decreasing as the potential increases.

In the first two chapters of the present paper no physical hypothesis to explain the phenomenon of gravitation was adopted; the only solitary assumption made was that the gravitational effect is propagated with a finite velocity. As in Relativity nothing can exceed the velocity of light, the assumption was natural enough. All the equations and the

formulæ were mere mathematical deductions by the application of the principle of the Döppler effect, the diminution of intensity with the inverse square of the distance and the shifting forward of the line of attraction in conformity with the principle of the aberration of light, when the two bodies, instead of being stationary, are in relative motion. In the present chapter, except Sec V., it is assumed that every particle of matter is emanating fine corpuscles called gravitons, smaller even than the light particles called radions. Cosmic radiations on the ultra violet side and the electro-magnetic waves on the infra red side are within the present range of our observation and we notice them. But the gravitons, finer even than cosmic rays, are just beyond our present range of perception, and so we do not observe them at all, but only detect their effect. This is just as Modern Physics assumes the existence of *neutrinos*, suggested by Pauli, although they have not yet been actually observed. There is absolutely no reason why the emanations from matter must be confined to only those radiations which our present day instruments can measure, as if Nature's limits are fixed by our capacity to observe them.

SECTION II

SELF-ACCELERATION

1. Newton's first law of motion was that a body will continue to move in a straight line with the same velocity for all time to come so long as it is not disturbed by an extraneous force. The same uniformity of motion is assumed in Relativity. But the theory of emission of gravitons necessarily leads to the conclusion that this is not so, when a body is both moving and simultaneously emitting gravitons. As a result of its motion, there would be less emission at the front than at the rear, the difference in the losses of momenta causing an automatic acceleration in the direction of motion.

2. Ordinarily radions and gravitons would leave their systems, when after innumerable revolutions they attain the parabolic velocity. (See Chapter II, Section VII, para 2). As explained in the Unified Theory the gravitons leave a body when they attain the limiting parabolic velocity D. But the stellar conditions owing to the prevalence of high temperatures are quite different from terrestrial conditions. If a physical explanation were wanted, it can be supposed that gravitons emerge as a result of a sudden explosion of a sub-atomic shell so that its fragments fly off with equal momentum in space along the radii. As the gravitons are the cause of gravitation and are not themselves subject to gravitation,

their velocity is unaffected by the gravitational attraction of the star or nebula

3. For purposes of this chapter it is not necessary to make any assumption as regards gravitons other than what Einstein has assumed for light and what de Sitter has inferred from his test of binary stars, *viz.*, that the velocity of gravitons in space is independent of that of its source. Thus, if a body be moving with velocity v, then two tiny fragments (*i.e.*, gravitons) of masses μ at the front and at the rear will emerge with momenta

$$\mu v + (\mu D - \mu v) = +\mu D$$

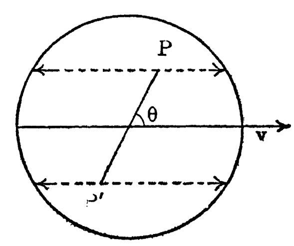
and

$$\mu v - (\mu D + \mu v) = -\mu D$$

These are independent of the original velocity v.

4. Thus gravitons leave a body with the limiting velocity D. And so each possesses a momentum μD. If the body from which it emerges is stationary, each of the gravitons on either side takes away a momentum $+\mu D$ and $-\mu D$ respectively from it, in which case the emission causes no acceleration as the net result is zero. But if the body were moving with velocity v in one direction, there would have been no effect on the remaining mass if the gravitons emerging in the same direction were to go off with momentum μv each. But each actually goes off with μD. And so the momentum that affects the remaining mass is $\mu(D - v)$ instead of $\mu.D$. There is thus a saving of momentum $\mu . v$. On the other hand, in the rear the graviton in order to acquire a velocity D in space has to take away a momentum $-\mu(D + v)$ because it had a velocity $+v$ along with the body. Hence the effect of the difference in the losses of momenta at corresponding points in the front and the rear is a net saving of $2\mu . v$, which is effective as a gain in the direction of the motion of the body ... ... (19·1)

5. If we take two corresponding points P and P′ inside a sphere which is moving with velocity v, and gravitons start from P and P′ with velocities $\pm$ D parallel to v, then those travelling in the direction of v will take a longer time to reach the surface and to leave the sphere than those travelling in the opposite direction. It is easy to see that the frequency of emission, *i.e.*, the losses of gravitons in the front would be diminished in the ratio $\left(1 - \frac{v}{D}\right)$ whereas that



at the rear will be increased in the ratio $\left(1 + \frac{v}{D}\right)$. If each graviton

carries away a momentum $\pm \mu D$, then the difference in the momenta of corresponding gravitons is $-2\mu D$. Hence the net gain to the remaining mass is $+2\mu D$ in the direction of motion......(19·1). It follows that an isolated body moving in space accelerates its motion.

6. Were it to be wrongly assumed, as in Relativity, that the velocity of the emerging graviton *with respect to* the body is constant and is the same on both sides, then the losses in the front and at the rear would be $\mu(D+v)$ and $\mu(D-v)$, the net result being a retardation of motion instead of acceleration.

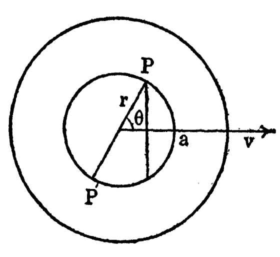
There is an equal fallacy in the supposition[17] that when a star is moving forwards the emitted radiation is rather heaped up in front and thinned out behind; and as radiation exerts pressure, the pressure will be stronger on the front than on the rear causing retardation. This would imply that old stars should have lower speeds than young stars, which is contrary to observation. As the speed of radiation is very much greater no part of the emitted radiation can really lag behind to exert any backward pressure at all.

Section III

THE COSMICAL CONSTANT

1. If all particles of a sphere emit gravitons in all directions, it is easy to see that the effect will be the same as if the entire mass were concentrated at the centre, and the gravitons were emitted from the centre. If a sphere were divided into a series of thin concentric spherical shells, the result is the same as if the mass of each shell were emitting gravitons from every point in the shell along its radial distance.

2. Let a spherical mass of radius a be moving with velocity v. Let n be the number of gravitons emanating from unit mass per unit time, μ and D the mass and velocity of each.



Consider a spherical shell of radii r and $r+dr$, and take its section which is at right angles to the motion of the sphere and subtends an angle 2θ at the centre. Let ρ be the density. Then the mass of a disc $= (2\pi r \sin\theta \, rd\theta \, dr\rho)$. The resolved velocity of the point P along the radius is $v\cos\theta$. The gravitons emerging radially from P and P′ would therefore yield a net momentum $2\mu v \cos\theta$, which resolved along the path of motion of the sphere gives $2\mu v \cos^2\theta$. Hence

the total gain in momentum in the direction of v per unit time

$$= \int_0^a \int_0^{\pi/2} 2\pi r \sin\theta \, r\rho . \, 2\mu v \cos^2\theta \, n \, d\theta \, dr.$$

$$= n\mu \frac{4\pi}{3} v\rho \frac{a^3}{3}$$

$$= \frac{Mv}{3} (n\mu), \text{ where M is the total mass of the sphere.}$$

Hence the acceleration caused to the sphere by its own motion is

$$\frac{d^2R}{dt^2} = \left(\frac{n\mu}{3}\right) v = \gamma \frac{dR}{dt} \quad \ldots \quad \ldots \quad \ldots \qquad (20{\cdot}1)$$

where γ is the Cosmical Constant.

The dimensions of γ are $= n\mu = \frac{1}{MT} \cdot M = \frac{1}{T} \cdot$

3. *Alternative method.* Let n be the number of gravitons emanating per unit mass per unit time, μ the mass of each, and D its velocity.

Then the number of gravitons emerging through a solid angle dw cause a loss of momentum $= nM \frac{dw}{4\pi} dt \; \mu D$, if the body be at rest.

But if the body is moving with velocity v, then at P, making an angle θ, the loss $= nM \frac{dw}{4\pi} dt. \; \mu \, (D - v \cos\theta)$. Similarly the loss at P′ $= -nM \frac{dw}{4\pi} dt. \; \mu(D + v \cos\theta)$. Resolving these two along the direction of motion, the combined gain $= 2 \, nM. \frac{dw}{4\pi} dt. \; \mu \, v \cos^2\theta$.

$$\text{Now} \int_0^{\pi/2} \cos^2\theta \, dw = \int_0^{\pi/2} \cos^2\theta \; 2\pi \; \sin\theta \; d\theta = \frac{2\pi}{3}$$

Therefore the real change in loss of momentum

$$= \frac{M}{3} (n\mu) \, v dt.$$

Hence the acceleration $= \frac{n\mu}{3} v$.

$$\therefore \frac{d^2R}{dt^2} = \frac{n\mu}{3} \frac{dR}{dt} = \gamma \frac{dR}{dt}$$

where $\gamma = \frac{n\mu}{3}$ is the Cosmical Constant $\quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20{\cdot}1)$

4. (i) If $\frac{d^2R}{dt^2} = \gamma \frac{dR}{dt}$, then $\frac{dR}{dt} = \gamma\ R+A_0$ ... ... (20·2)

Multiplying by $e^{-\gamma t}$ we get $\frac{d}{dt}(Re^{-\gamma t}) = A_0e^{-\gamma t}$

$\therefore\ R = -\frac{A_0}{\gamma} + B_0\, e^{\gamma t}$.

This can be written as $R = Ae^{\gamma t} + B$ ... ... (20·3)

(ii) Now if $R = Ae^{\gamma t} + B$.

$$\text{then } \frac{dR}{dt} = \gamma Ae^{\gamma t} = \gamma\,(R-B)$$

$$\frac{d^2R}{dt^2} = \gamma^2 Ae^{\gamma t} = \gamma^2(R-B) = \gamma\,\frac{dR}{dt}.$$

If $R = R_0$ and $v=v_0$ when $t=0$, then

$$\frac{dR}{dt} = \gamma\ R + (v_0-\gamma R_0) \quad \dots \quad \dots \quad \dots \quad (20{\cdot}4)$$

$$\text{and } R = \frac{v_0}{\gamma}\, e^{\gamma t} + \left(R_0-\frac{v_0}{\gamma}\right) \dots \quad \dots \quad \dots \quad (20{\cdot}5)$$

SECTION IV

PROPORTIONALITY TO DISTANCE LAW

1. *In relativity* for a particle at rest, $\frac{d^2r}{ds^2} = \frac{1}{3}\lambda r\ (1-\frac{1}{3}\lambda r^2)\left(\frac{dt}{ds}\right)^2$

$$\text{and } \left(\frac{dt}{ds}\right)^2 = \frac{1}{(1-\frac{1}{3}\lambda r^2)} \quad \dots \quad \dots \quad \dots \quad (21{\cdot}1)$$

$$\text{Hence } \frac{d^2r}{ds^2} = \frac{1}{3}\lambda r.$$

As light cannot cross the barrier, one is limited to the region where $(1-\frac{1}{3}\lambda r^2)$ is positive. Hence there are no reasonable conditions which encourage a motion towards the origin. The force of repulsion acts away from the origin; and the velocity also is one of recession.

$$2\,\frac{dr}{ds}\cdot\frac{d^2r}{ds^2} = \frac{1}{3}\lambda r\, 2\,\frac{dr}{ds}.$$

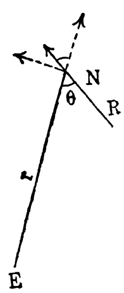
$$\therefore \left(\frac{dr}{ds}\right)^2 = \frac{1}{3}\lambda r^2 + A \text{ and so } \frac{dr}{ds} = +\sqrt{\tfrac{1}{3}\lambda r^2 + A} \qquad (21{\cdot}2)$$

The velocity is proportional to distance only if the constant is taken to be zero. Relativity is utterly unable to explain how some nebulæ, including the great Nebula in Andromeda, can be approaching.

As both the acceleration and the velocity are positive when measured away from the origin and most nebulæ are moving away from the earth with velocities proportional to their distances, it would seem that the earth is the centre of the explosion—an unreasonable conclusion.

The stars on the opposite sides of a diameter of the Galactic system are 250,000 light years apart and as one light year = $9{\cdot}4 \times 10^{17}$ cm., their distances are about $2{\cdot}4 \times 10^{24}$ cm. And yet their velocities do not scatter them away.

2. (i) In the *New Theory* $\frac{d^2R}{dt^2} = \gamma \frac{dR}{dt}$, where R may be measured in any direction and from any origin. The acceleration is proportional to the velocity and is in the direction of the velocity. Their components along the line of sight will have the same proportion. And the nebula can be either receding or approaching. The component along the line of sight is $\frac{dr}{dt} = \frac{dR}{dt}\cos\theta$, which is observable by means of the spectral shift. The component normal to it is $\frac{rd\theta}{dt} = \frac{dR}{dt}\sin\theta$ and is not observable.



(ii) It has been shown that the self-acceleration is proportional to velocity and is along the direction of the motion. So if a nebula were moving in a direction making an angle θ with the line of sight from the earth and R be measured along that direction and r along the line of sight then

$$\frac{d^2R}{dt^2} = \gamma.\frac{dR}{dt} \therefore \frac{d^2R}{dt^2}.\cos\theta = \gamma.\frac{dR}{dt}.\cos\theta$$

Hence $$\frac{d^2r}{dt^2} = \gamma.\frac{dr}{dt} \quad \dots \quad \dots \quad \dots \quad (21{\cdot}3)$$

Here γ is a universal constant. On integration the equation gives the approximate proportionality to distance law for the component velocity along the line of sight.

(iii) We thus get a Cosmological Principle that the relation of the acceleration and velocity of a moving body presents the same picture to an observer, no matter where he is placed. The earth need not be at the centre of the Universe and we may be looking at a nebula from any position in space, the ratio of the acceleration to velocity still remains the same, and they can be both positive or both negative. In Relativity a

nebula must be moving in a direction away from the earth as origin, whereas in the New Theory it can be moving in any direction; it is only the resolved component of its velocity along the line of sight which causes the spectral shift.

(iv) It was shown in Chapter III Section VI that loss of frequency of light can be due to several causes and would be proportional to distance. If the loss in frequency due to causes other than recession be $-\triangle\nu = g.r$,

$$\text{then } \triangle\lambda = \frac{g.\lambda^2}{c}.\, r, \text{ nearly.}$$

Now, if the nebula is receding with velocity v then, $(\nu+\delta\nu) = (1-\frac{v}{c})\,\nu$

$$\therefore\ \lambda+\delta\lambda = (1+\frac{v}{c})\,\lambda. \quad \text{Hence } \delta\lambda = \frac{v}{c}.\lambda. \qquad \dots \qquad (21{\cdot}4)$$

Hence if $d\lambda$ be the observed shift, then the real shift due to recession

$$\text{is } \delta\lambda = d\lambda - \triangle\lambda = d\lambda - \frac{g\lambda^2}{c}.\, r. \qquad \dots \qquad \dots \qquad (21{\cdot}5)$$

Accordingly $v = \frac{\delta\lambda}{\lambda}\, c = (d\lambda - \frac{g\lambda^2}{c}.\, r)\, \frac{c}{\lambda} = (\gamma - g\lambda).\, r$

where by observation $\frac{\triangle\lambda}{\lambda}.\, c = \gamma.r.$ ... ... (21·6)

Thus the real recessional velocities of the nebulæ may be considerably smaller than the observed recessional velocities.

SECTION V

SURFACE RADIATION

1. Even if the emission of gravitons from the entire mass be not assumed, and it be only supposed that light particles are emitted from an outer shell of radii a_1 and a_2 then from Section III §2 the total gain of momentum from the light radiation alone will be

$$= \int_{a_2}^{a_1} \int_0^{\frac{\pi}{2}} 2\pi r \sin\theta.\, r.\, \rho.\, 2\mu v \cos^2\theta.\, n.\, d\theta.dr$$

$$= 4\pi\mu\, n.\, v \int_{a_2}^{a_1} \int_0^{\frac{\pi}{2}} \cos^2\theta \sin\theta\; r^2\rho.\; dr\; d\theta. \qquad \dots \qquad \dots \qquad (21{\cdot}7)$$

(i) Assume that density is uniform, then ρ is constant.

$$\therefore \text{ the gain } = 4\pi \frac{\mu n.}{3} . v. \rho. \frac{a_1{}^3 - a_2{}^3}{3}$$

$$= (\tfrac{4}{3} \pi a_1{}^3. \rho). \frac{n\mu v}{3} (1 - \frac{a_2{}^3}{a_1{}^3})$$

$$= \text{M}. \frac{n\mu}{3}. v (1 - \frac{a_2{}^3}{a_1{}^3})$$

Hence $$\frac{d^2R}{dt^2} = \gamma'. v. = \gamma' \frac{dR}{dt}$$

(ii) If $\rho = \frac{m}{r}$, where m is constant, then

$$\text{the gain } = (2\pi m. a_1{}^2). \frac{n\mu}{3}. v (1 - \frac{a_2{}^2}{a_1{}^2})$$

$$\text{But the mass } = \int_0^{a_1} \frac{m}{r} \pi. 4\pi r^2 dr = 2\pi m. a_1{}^2,$$

Hence again $$\frac{d^2R}{dt^2} = \gamma''. v. = \gamma''. \frac{dR}{dt}.$$

(iii) If $$\rho = \frac{m}{r^2},$$

then the gain $$= \text{M}. \frac{n\mu}{3}. v (1 - \frac{a_2}{a_1})$$

and so on.

Hence the same relation between the acceleration and the velocity holds in the case of the surface radiation also, only the constants are different.

2. If a nebula has mass M, radius a and radiation σ per unit surface area per second, then $\frac{4\pi a^2. \sigma \times 3 \times 10^{10}}{\text{M}} = \gamma. v$, if light radiations alone were effective. The substitution of the observed values will test whether this is possible. The time when the nebulæ left our galactic system would be increased considerably, if radiations from an outer shell only are the cause of its acceleration.

SECTION VI

THE RECEDING AND APPROACHING NEBULÆ

1. The galactic system is estimated to have the weight of about 18×10^{10} suns, and the distance of the Great Nebula (M. 31 in Andromeda) is 9×10^5 light years.

If the nebula were just on a parabola round the G. S.

$$v_1{}^2 = \frac{2\text{ G M}}{\text{R}} = \frac{2 \times 18 \times 10^{10} \times 1{\cdot}98 \times 10^{33} \times 6{\cdot}673 \times 10^{-8}}{9 \times 10^5 \times 9{\cdot}463 \times 10^{17}} = 5{\cdot}52 \times 10^{13}.$$

But the observed velocity after making deduction for the galactic rotation is $v_2{}^2 = (20 \times 10^5)^2 = 4 \times 10^{12}$, which would be within the limiting velocity. Hence the nebula is still a part of the galactic system and is revolving round it in a nearly elliptic orbit.

2. It is submitted that the weight of the galactic system has been much under-estimated, because it is difficult to estimate the total number of the invisible stars and the total weight of clouds of matter at the outer fringe of the Milky Way. It is also probable that the distances of nebulæ are over-estimated. Lastly, the real shift is less than the apparent shift. Thus the limiting velocity for the effective range of the galactic system may be much larger, and so a very large majority of the nebulæ may still be parts of our system.

3. All the nebulæ, which have greater velocities, are going away on hyperbolas, hence the resolved components of their velocities always show recession. This is why no nebula with a velocity greater than the limiting velocity is found to be approaching. These may pass on to some other galactic system like electrons passing on from one atom to another.

On the other hand, all the nebulæ having velocities less than the limiting value are still moving in ellipses round the G. S., and so some are seen to be approaching and some receding according to their position in the orbits.

4. Another possible, but improbable, hypothesis can be that the approaching nebulæ left some other galactic system, and so happen to be coming towards our galactic system, just as nebulæ leaving our galactic system may by chance happen to pass close by another galactic system and be caught by it.

Section VII

1. When the acceleration proportional to velocity along the path of motion is taken into account an additional term $\frac{d^2s}{dt^2} = \gamma \frac{ds}{dt}$ has to be included in the equations of motion, which in place of (5·41) and (5·7) take the following forms :—

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = \frac{\mu}{r^2}\,\frac{r}{\mathrm{D}}\,\frac{d\theta}{dt} + \gamma r\frac{d\theta}{dt} \quad \ldots \quad \ldots \quad (22·1)$$

$$\text{and } \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{\mu}{r^2} - \frac{3\mu}{\mathrm{D}^2}\,\frac{1}{r^2}\left(r^2\frac{d\theta}{dt}\right)^2 + \frac{3\mu}{\mathrm{D}}\,\frac{1}{r^2}\,\frac{dr}{dt} + \gamma\frac{dr}{dt} \quad (22·2)$$

It is noteworthy that the additional terms are not only independent of the mass of the influenced body, but also of that of the influencing body. Hence in the solar system, where the sun has a huge mass, the effect of the additional terms is almost negligible. It is only after a comet has started on a parabolic path, that the influence of the sun diminishes and the effect of self-acceleration increases. The same is of course true of a nebula leaving the galactic system by reaching the parabolic velocity.

2. It is easy to see that if the galactic system and a nebula forming part of it, were both moving through space with the same velocity, the self-acceleration of both due to their velocity will always remain equal. The same would be the case even after the nebula has separated from the galactic system. But its recessional velocity, as seen from the galactic system, would be caused exclusively by the additional acceleration due to its relative motion. But if the nebula is emitting matter, *e.g.*, light at a higher rate than the galactic system it will have a greater acceleration. In that event the nebulæ that have left our system will be more crowded in the direction of the motion. It is actually found that the density of the nebulæ distribution to the north of the Milky Way is greater than in the south (Shapley).[18] So the galactic system is moving towards the North side.

SECTION VIII

THE STABLE UNIVERSE

1. The extra-galactic island nebulæ are like miniature galactic systems, in all respects similar to ours, possessing numerous stars, comets and dust clouds. These miniature galactic systems are or were revolving round our galactic system as nucleus, or more accurately round the common centre of gravity, just as planets revolve round the sun.

2. Their orbital velocity was gradually increased owing to the net attractive pull as well as their self-acceleration, till the more distant ones attained the limiting velocity equal to $\frac{2M}{R_0}$, when they started on a parabolic path not likely to return to the galactic system. R_0 is of course different for each nebula.

3. These are travelling on the arms of straightened parabolas, at different inclinations to the line of sight. But only their radial velocity is observable because of the shift of the spectral lines, as light alone is a measuring instrument for such huge distances. Their transverse velocity cannot be measured, as the distance is too large and the change of angle

too small. As the observed velocities of most of these nebulæ are much greater than the limiting velocity, they left the galactic system long ago.

4. But the nearer nebulæ have less than the limiting velocity, and so they are still revolving round our galactic system in huge orbits. Some are approaching and some receding according to their positions in their nearly elliptic orbits.

5. The universe is perfectly stable, and is not exploding. Only some nebulæ, which were at one time parts of our galactic system, have left it on parabolic paths. They will travel away until they are caught by some other huge galactic system like our own, which owing to its distance is not yet visible to us. Their passage would be like electrons from atom to atom.

6. All such galactic systems would form parts of a bigger super-galactic system. Such super-galactic systems would in their turn form parts of a Meta-galactic system; and so on, beyond the comprehension of the human mind. They all have the same features as the solar system, though on a much more gigantic scale. Space is not at all so finite and surprisingly small as Einstein conceives it to be. But howsoever far we see, space is strictly three-dimensional; and there is absolutely no evidence that the spatial phenomena are anything but three dimensional. A four-dimensional continuum, obtained by confusing time with space, is not at all called for to explain the Universe.

CHAPTER V

Special Relativity

SECTION I

COMMON TIME AND DISTANCE

1. Newton, starting with common absolute space and time for all moving bodies, assumed that dynamically the relative velocity between two bodies is exactly the same as if either were reduced to rest, and the other were moving with the difference in their velocities. If two bodies were moving with absolute velocities u and v', and v be their relative velocity, then $v = v' - u$. Newton would have been right if v' and u could be measured instantaneously by means of a messenger travelling with infinite velocity or by an omniscient superman.

2. In Einstein's Relativity it is considered impossible to have a common time between two bodies. Each body is supposed to keep its own separate time, and there is no method by which both can measure exactly the same time, except by the impossible assumption that light from the first to the second takes the same time as light from the second to the first. "We have hitherto an A-time, and a B-time, but no time common to A and B. This last time (*i.e.*, common time) can be defined, if we establish by definition that the time which light requires in travelling from A to B is equivalent to the time which light requires in travelling from B to A." (Einstein)[19] But in reality the times taken by light are not equal, as $\frac{r}{c-v'} \neq \frac{r}{c+u}$ for all values of u and v'.

3. But there is a method according to which two moving bodies can measure *exactly* the same time and can calculate *exactly* the same distance between them. This is the method of Reflection or Double journey. A messenger is sent out from A to B and returns as soon as it meets B and then meets A and the time taken by the messenger is measured by A. Another messenger is sent by B to A and after meeting A returns to B and meets B, and the time taken by the messenger is measured by B. If the two messengers travel with the same absolute velocity D, it can be shown dynamically that the times measured by A and B separately are exactly the same. It can also be shown dynamically that the distances between A and B as calculated by both separately are exactly the same.

4. But both A and B labour under a mistake. Each regards himself as if he were at rest and the other moving away from him with the difference in their absolute velocities between them. Or he wrongly assumes with Newton that no matter what his own absolute velocity may be, he gets the correct result by reducing himself to rest and taking the difference between the two velocities as the relative velocity. But in point of fact, the relative velocity between them depends not only on the difference in their absolute velocities, but also to some extent on the absolute velocities themselves. The assumption that only the difference in the absolute velocities comes into play causes an error in the Newtonian law, for which a correction is necessary when account is taken of their absolute velocities.

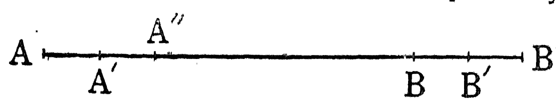
5. Newton's absolute space and time were philosophically conceivable and mathematically workable, though not actually measurable. The relative velocity between two bodies moving uniformly was in absolute space and time, an exact, determinate quantity, if their velocities could be

measured instantaneously. Einstein has rejected absolute space and time, simply because they are not actually measurable, and regards relative velocity as a measurable quantity. But if the absoluteness of space and time be rejected, then relative velocity also becomes an uncertain quantity. If relative velocity only means relative velocity as actually observed and we go exclusively by measurements only, then the relative velocity between two bodies would depend on the particular method of measurement chosen. It will be shown in the next paper that the value would vary as the method is changed. Further, a personal equation will come in according as one body or the other is taken as source or observer. But the relative velocity between the two moving bodies ought to be the same, quite independent of any such personal equation.

SECTION II

THE REAL AND APPARENT VELOCITIES

1. Let A and B be moving with velocities u and v' respectively. Let a messenger be sent out from A to B, overtake it at B′ and return to A and meet it at A″ and let the messenger travel with the velocity D; and let AB $= r$. Let A′ be the position of A when B is at B′.



Then the real time taken from A to B′ + the real time taken from

$$\text{B}' \text{ to } \text{A}'' = \frac{\text{AB}}{\text{D}-v'} + \frac{\text{A}'\text{B}'}{\text{D}+u} = \frac{r}{\text{D}-v'} + \frac{r + \frac{r}{\text{D}-v'} \cdot v' - \frac{r}{\text{D}-v'} \cdot u}{\text{D}+u}$$

$$= \frac{2.\,\text{D}.\,r}{(\text{D}-v')\,(\text{D}+u)}. \qquad \ldots \quad \ldots \quad \ldots \quad (23{\cdot}1)$$

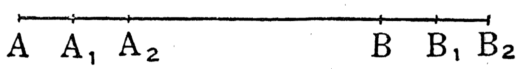
Thus the real time taken for the double journey is $t_1 + t_2$

$$= \frac{2.\,\text{D}.\,r}{(\text{D}-v')\,(\text{D}+u)}.$$

The real distance between A″ and B″ after the messenger has returned is A″B″ = A″ B′ + B′ B″

$$= \frac{r + \frac{r}{\text{D}-v'}\, v' - \frac{r}{\text{D}-v'}\, u}{\text{D}+u} . \text{D} + \frac{r + \frac{r}{\text{D}-v'}\, v' - \frac{r}{\text{D}-v'}\, u}{\text{D}+u} . v'$$

$$= \frac{r\,(\text{D}-u)\,(\text{D}+v')}{(\text{D}+u)\,(\text{D}-v')} \qquad \ldots \quad \ldots \quad \ldots \quad (23{\cdot}2)$$



Now suppose that a message is sent out from B, reaches A at A_1 and returns to B at B_2, and that B is at B_1 when A is at A_1.

Then the real time taken by the messenger from B to A_1 + the real time taken from A_1 to B_2

$$= \frac{AB}{D+u} + \frac{A_1 B_1}{D-u} = \frac{r}{D+u} + \frac{r + \frac{r}{D+u} \cdot v' - \frac{r}{D+u} \cdot u}{D-v'}$$

$$= \frac{2\, D.\, r}{(D-v')\,(D+u)} \qquad \ldots \qquad \ldots \qquad (23{\cdot}3)$$

Thus the real time $t_1 + t_2$ is the same as (23·1).

The real distance between B_2 and A_2 after the messenger has returned to B is $B_2 A_2 = B_2 A_1 - A_1 A_2$

$$= \frac{r + \frac{r}{D+u} \cdot v' - \frac{r}{D+u} \cdot u}{D-v'} \,.\, D - \frac{r + \frac{r}{D+u} \cdot v' - \frac{r}{D+u} \cdot u}{D-v'} \,.\, u$$

$$= \frac{r\,(D-u)\,(D+v')}{(D+u)\,(D-v')} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (23{\cdot}4)$$

Thus the real distance between B_2 and A_2 is the same as that between A'' and B'' in (23·2)

It follows that both A and B measure the same time and the same distance if they both employ messengers travelling with the same velocity.

2. But A regards himself at rest, and assumes that B is moving away from him with velocity $(v' - u)$.

Then the apparent time as measured by A when a messenger sent out from A returns from B to A, is $2\,t$

$$= \frac{r}{D-(v'-u)} + \frac{r}{D-(v'-u)} = \frac{2.\, r}{D-v'+u} \qquad \ldots \qquad (23{\cdot}5)$$

for, according to him the messenger will take the same time to overtake B as to return to A after overtaking B., as A is supposed to be at rest.

The apparent distance between A and B'' according to him is

$$A\, B'' = r + \frac{2\, r}{D-v'+u} \cdot (v'-u) = \frac{D+v'-u}{D-v'+u} \,.\, r \qquad \ldots \qquad (23{\cdot}6)$$

On the other hand, B regards himself as at rest, and assumes that A is moving away from him with velocity $(v'-u)$ in the opposite direction.

Hence the apparent time measured by B is the same and

$$= \frac{2\,r}{D-v'+u}.$$

Similarly the apparent distance between B and A$''$ is the same and

$$= \frac{D+v'-u}{D-v'+u} \cdot r.$$

Thus both A and B measure exactly the same apparent time and distance.

3. Therefore owing to the simultaneous motions of both the bodies instead of a distance r we have a real distance (23·2)

$$\frac{(D-u)\,(D+v')}{(D+u)\,(D-v')} \cdot r$$

and instead of time $\frac{2r}{D}$ we have a real time (23·1)

$$\frac{2\,Dr}{(D-v')\,(D+u)}$$

while instead of r we have an apparent distance (23·6)

$$\frac{D+v'-u}{D-v'+u} \cdot r$$

and instead of time $\frac{2r}{D}$ we have an apparent time (23·5)

$$\frac{2r}{D-v'+u}.$$

Similarly $(r+dr)$ becomes $\frac{(D-u)(D+v')}{(D+u)(D-v')}\,(r+dr)$

and $\frac{D+v'-u}{D-v'+u} \cdot (r+dr)$ respectively,

and $\frac{2(r+dr)}{D}$ becomes $\frac{2D\,(r+dr)}{(D-v')\,(D+u)}$,

and $\frac{2\,(r+dr)}{D-v'+u}$ respectively.

Hence an infinitesimal distance dr becomes a real distance

$$= \frac{(D-u)(D+v')}{(D+u)(D-v')}.dr$$

and an apparent distance $= \frac{D+v'-u}{D-v'+u}\cdot\ dr$

while an infinitesimal time $\frac{2dr}{D}$ becomes a real time

$$= \frac{2Ddr}{(D-v')(D+u)}$$

and an apparent time $= \frac{2dr}{D-v'+u}$.

It follows that the *real* relative velocity $(v'-u)$ changes in the ratio

$$= \left\{\frac{(D-u)(D+v')}{(D+u)(D-v')} \div \frac{D^2}{(D-v')(D+u)}\right\} = \frac{(D+v')(D-u)}{D^2} \quad \dots \quad (23\cdot7)$$

and the *apparent* relative velocity v changes in the ratio.

$$= \left\{\frac{D+v'-u}{D-v'+u} \div \frac{D}{D-v'+u}\right\} = \frac{(D+v'-u)}{D} \quad \dots \quad \dots \quad (23\cdot8)$$

Thus instead of Newton's formula $v=v'-u$ we have

$$\frac{v}{v'-u} = \frac{D(D+v'-u)}{(D+v')(D-u)}$$

$$= \frac{1+\frac{v'}{D}-\frac{u}{D}}{(1+\frac{v'}{D})(1-\frac{u}{D})} \quad \dots \quad \dots \quad \dots \quad (23\cdot9)$$

$$= \frac{1+\frac{v'}{D}-\frac{u}{D}}{1+\frac{v'}{D}-\frac{u}{D}-\frac{v'u}{D^2}}$$

$$= \frac{1}{1-\frac{v'u}{D^2}+\frac{v'^2u}{D^3}-\frac{v'u^2}{D^3}+\dots} \quad \dots \quad \dots \quad (23\cdot10)$$

$$= \frac{1}{1-\frac{v'u}{D^2}} \text{ nearly} \quad \dots \quad \dots \quad \dots \quad \dots \quad (23\cdot11)$$

This corresponds in form with the famous formula assumed by Einstein for the addition of relative velocities in special Relativity with light as the messenger, which is not rigorously true but only approximately so. The correspondence with Einstein's formula comes in because the relative velocities of a moving point with reference to two moving systems correspond with the absolute velocities of those systems when the third is a point at rest in space.

4 If we put v' for v, v for v' and $-u$ for u

then $$\frac{v'}{v+u} = \frac{1+\frac{v}{D}+\frac{u}{D}}{\left(1+\frac{v}{D}\right)\left(1+\frac{u}{D}\right)} = \frac{1+\frac{v}{D}+\frac{u}{D}}{1+\frac{v}{D}+\frac{u}{D}+\frac{vu}{D^2}} \quad \ldots \quad (23\cdot12)$$

$$= \frac{1}{1+\frac{vu}{D^2}-\frac{v^2u}{D^3}-\frac{vu^2}{D^3}+\ldots} \quad \ldots \quad \ldots \quad \ldots \quad (23\cdot13)$$

$$= \frac{1}{1+\frac{vu}{D^2}} \text{ nearly} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23\cdot14)$$

The significance of this converse formula, which may not be apparent, will be shown later.

5. If u and v' be not the absolute velocities in space, but velocities relative to a point moving with an unknown absolute velocity x, then the formulae can be easily seen to be

$$\frac{v}{v'-u} = \frac{1+\frac{v'+x}{D}-\frac{u+x}{D}}{\left(1+\frac{v'+x}{D}\right)\left(1-\frac{u+x}{D}\right)} \quad \ldots \quad \ldots \quad (23\cdot15)$$

and $$\frac{v'}{v+u} = \frac{1+\frac{v+x}{D}+\frac{u+x}{D}}{\left(1+\frac{v+x}{D}\right)\left(1+\frac{u+x}{D}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (23\cdot16)$$

6. If the two points A and B be moving with the same velocities then $v' = u$.

Hence from (23.1) the real time is $t_1+t_2 = \frac{2\,D\,r}{(D^2-u^2)}$.

" (23·2) " distance $= r$.

" (23.5) the apparent time is $2t = \frac{2r}{D}$.

" (23.5) " distance is $= r$.

And although $v'-u=0$ and therefore $v=0$, the ratio

$$\frac{v}{v'-u}=\frac{1}{1-\frac{u^2}{D^2}}.$$

The First Universal Principle

The relative velocity v between two bodies moving with velocities u and v', measured by employing a messenger travelling with velocity D in a to-and-fro journey, is given by the formula

$$\frac{v}{v'-u}=\frac{1+\frac{v'}{D}-\frac{u}{D}}{\left(1+\frac{v'}{D}\right)\left(1-\frac{u}{D}\right)}$$

The Corollary.—When two bodies are moving with the same constant velocity u in the same direction, the ratio of their apparent and real relative velocities, measured by means of a messenger travelling with velocity D and performing the double journey, is not unity but $=\frac{1}{1-\frac{u^2}{D^2}}$.

7. *The transformation formulae.*—It is assumed in Relativity that a point P has coordinates (x, y, t) with reference to a system S and coordinates (x', y', t') with reference to a system S$'$ and that

$$\left.\begin{array}{r} x=0 \text{ and } x'=\ \ u_0t' \\ \text{and } x'=0 \text{ and } x'=-u_0t \end{array}\right\} \text{ simultaneously,}$$

where u_0 is the difference of their velocities. So that these assumptions involve the fallacies (1) that their different coordinates can be read off instantaneously, whereas if a messenger be employed he will take time to go from one to the other and back, and (2) that these coordinates are measured when the two systems are alternately at rest, which is not the true fact.

As approximations the assumptions are true, but not rigorously.

(1) The true transformations for a double journey are with the help of (23·2) given by the following set of equations :—

For the origins, $x=0$

$$\text{and } x'=\frac{(D-u)(D+v')}{(D+u)(D-v')}.(v'-u).t'=ku_0 \qquad \ldots \qquad (23\cdot17)$$

Also $\qquad x'=0$

and $x = -\dfrac{(D-u)\,(D+v')}{(D+u)\,(D-v')}.\,(v'-u)\,t = -ku_0$ ... ... (23·18)

where $k = \dfrac{(D-u)\,(D+v')}{(D+u)\,(D-v')}$ and $u_0 = v' - u.$,

and t and t' are supposed to be different and represent the whole times taken in the two systems. In reality $t=t'$.

Hence generally, $\left.\begin{aligned} \alpha\,x &= x' - ku_0\,t' \\ \beta\,x' &= x + ku_0\,t \end{aligned}\right\}$ where α and β are some constants.

And therefore $ku_0 t = \beta x' - x = \beta x' - \dfrac{x' - ku_0 t'}{\alpha}$

$$\therefore\ \alpha t = \frac{\alpha\beta - 1}{ku_0}.\,x' + t'.$$

Also as $ku_0 t' = x' - \alpha.x = \dfrac{x + ku_0.t}{\beta} - \alpha.x$

$$\therefore\ \beta.t' = \frac{(1-\alpha\beta)}{ku_0}.\,x + t.$$

(2) If V and V′ be the velocities of a point relative to the two systems measured instantaneously,

then $$V = \frac{x}{t} = \frac{x' - ku_0.\,t'}{\dfrac{\alpha\beta - 1}{ku_0}\,x' + t'} = \frac{V' - ku_0}{1 - (1-\alpha\beta).\dfrac{V'}{ku_0}} \quad \dots \quad (23·19)$$

and $$V' = \frac{x'}{t'} = \frac{x + ku_0.t}{\dfrac{(1-\alpha\beta)}{ku_0}.\,x + t} = \frac{V + ku_0}{1 + (1-\alpha\beta).\dfrac{V}{ku_0}} \quad \dots \quad \dots \quad (23·20)$$

(3) As an approximation, we can put $k = 1$ and then the assumptions in Relativity are nearly true if S and S′ can be supposed to be alternately at rest.

The transformations then become

and $$\left.\begin{aligned} \alpha x &= x' - u_0 t' \\ \beta x' &= x + u_0 t \end{aligned}\right\} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (23·21)$$

Therefore $$V = \frac{x}{t} = \frac{x' - u_0.t'}{\dfrac{\alpha\beta - 1}{u_0}.x' + t'} = \frac{V' - u_0}{1 - (1-\alpha\beta).\dfrac{V'}{u_0}} \quad \dots \quad \dots \quad (23·22)$$

and $$V' = \frac{x'}{t'} = \frac{x + u_0\,t}{\dfrac{1-\alpha\beta}{u_0}.\,x + t} = \frac{V + u_0}{1 + (1-\alpha\beta).\dfrac{V}{u_0}} \quad \dots \quad \dots \quad (23·23)$$

SECTION III

THE POSTULATES OF RELATIVITY

1. From (23.11) and (23.14) we have the approximate formulae which are not rigorously true:—

$$v = \frac{v'-u}{1-\frac{v'u}{c^2}} \quad \text{and} \quad v' = \frac{v+u}{1+\frac{v'u}{c^2}}.$$

If we make the wrong assumption, as in Relativity, that they are rigorously true for all values of v and v' then

if we put $v' = c$, we get $v = c$

and if we put $v = c$, we get $v' = c$.

This gives the false result that the velocity of light is absolute and finite velocities added to it or subtracted from it make no difference to it.

2. If we apply the above result to the formulae (23.22) and (23.23) we get the approximate results:—

$$c = \frac{c-u_0}{1-(1-\alpha\beta)\frac{c}{u_0}} \quad \text{and} \quad c = \frac{c+u_0}{1+(1-\alpha\beta)\frac{c}{u_0}}.$$

Both of these give $\alpha\beta = 1 - \frac{u_0^2}{c^2}$ . . . . . (23·24)

3. Now if besides making the two assumptions in Section II. § 6, we further wrongly assume, as in Relativity, that the two systems S and S′ are perfectly equivalent, then by putting x' for x, and t' for t and $-u_0$ for $+u_0$ in (23.21) we get

$$\alpha . x = x' - u_0 . t'$$

and therefore $\alpha . x' = x + u_0 . t$

Hence $\alpha = \beta = \sqrt{1-\frac{u_0^2}{c^2}}$, which is the factor in Lorentz transformations.

4. These postulates then lead us on irretrievably to the queer results in Special Relativity that

(1) length contracts,

(2) time extends,

and (3) mass increases, with velocity.

(See Max Born's Relativity [20] pp. 208, 209 and 228.)

5. With the help of these strange transformations, various experimental results like Fresnel's convection formula, Michelson and Morley's

experiment, Bucherer's experiment, Sommerfeld's fine structures of spectral lines are sought to be explained.

6. But there is in reality no need to assume these unconvincing postulates or depend on the extraordinary results following from them, for as shown hereafter all the experiments can be easily explained on a simple dynamical principle which is of universal application.

SECTION IV

THE PHYSICAL METHOD

It was pointed out in Chapter I, section IV, that if instead of Newton's assumption that force acts instantaneously, *i.e.*, that it travels with infinite velocity, a finite velocity D is attributed to the propagation of gravitational force, a correction is necessary for Newtonian Dynamics. That correction was shown to be the same as the result of the compounding of two finite velocities, *viz.*, that if a body were moving with velocity v in a direction making an angle θ with the apparent direction of the force, then if the force be represented by a velocity of propagation D, it would no longer act effectively in the same direction in which it would have acted if the body had been stationary, but its actual action would be the same as if the body were stationary and the direction of the force were shifted forward by an angle α given by the ratio applicable to the compounding of the two velocities $\frac{\sin\alpha}{\sin\theta}=\frac{v}{D}$ (see figures on pp. 7 and 8)

If the effect of the motion of the body is merely to shift forward the direction of the field of force without affecting its intensity, then the component force which we observe along its direction would be D cos α. It follows that the correction to be introduced in Newtonian Mechanics in order to make it applicable to forces propagated with a finite velocity D, when applied to bodies moving with velocity v making an angle θ with the apparent direction of the force, is simply to reduce the intensity of that force by that factor. This simple correction to Newtonian Mechanics gives us all the results of Einstein's Relativity without his extraordinary assumptions. If ϕ be the angle between the real direction of the force and the direction of the body, then the angle of the shift is given by the equation $\frac{\sin\alpha}{\sin(\phi-\alpha)}=\frac{v}{D}$. Hence if the force be perpendicular $\alpha=\tan^{-1}\frac{v}{D}$.

For all heavenly bodies $\frac{v}{D}$ is small, and so for all practical purposes

$\tan \alpha = \sin \alpha = \frac{v}{D}$ and therefore $\cos \alpha = \frac{1}{\sqrt{1+\frac{v^2}{D^2}}} = \sqrt{1-\frac{v^2}{D^2}}$ nearly.

This principle of aberration which is merely the necessary result of the compounding of two dynamical velocities is of general application, and may be stated as

The Second Universal Principle

Wherever there is a uniform field of force which takes time to act, and can therefore be represented by a velocity of flow D, no matter whether of gravitation or radiation or electric or magnetic field, then its action on a body moving uniformly with velocity v inclined at an angle ϕ to the real direction of the flow, is exactly the same as if the body were stationary and the direction of the flow were shifted forward by an angle α given by the ratio applicable to the compounding of the two velocities $\frac{\sin \alpha}{\sin (\phi-\alpha)} = \frac{v}{D}$; so that when the angle is a right angle, $\alpha = \tan^{-1}\frac{v}{D} = \sin^{-1}\frac{v}{D}$ nearly.

In most experimets with light it is only the apparent direction of the light that is seen, and so when the motion is perpendicular to the apparent direction of light, $\theta = \frac{\pi}{2}$ and the factor is $\sqrt{1+\frac{v^2}{D^2}}$ exactly.

Thus if D represent the velocity of the flow, it follows by the simple process of the compounding of velocities that the effect of the motion of the body is merely to decrease the magnitude of the force along its apparent direction by the factor

$$\cos\left(\sin^{-1}\frac{v}{D}\right) = \sqrt{1-\frac{v^2}{D^2}}.$$

This process gives a physical explanation of the illusory character of the absoluteness and the maximum speed of light.

Section V

MINKOWSKI'S EQUATION

1. If as a first approximation we start with the assumption that the intensity of the force is reduced by the factor $\cos \alpha = \sqrt{1-\frac{v^2}{D^2}}$ along its direction, then we can by parity of reasoning deduce that the effective

velocity c_1 of light on a body moving with velocity v at right angles to the direction of the light will be given by the ratio

$$c_1 = c. \cos \alpha = c\sqrt{1 - \frac{v^2}{c^2}}$$

$$\therefore \quad c_1{}^2 dt^2 = c^2 dt^2 . \left(1 - \frac{v^2}{c^2}\right) = c^2 dt^2 \left(1 - \frac{1}{c^2} \cdot \frac{(dr)^2}{(dt)^2}\right)$$

$$= c^2 dt^2 - dr^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2)$$

$$= ds^2$$

if the projection of the effective path of light along the real direction be taken to be ds.

This is the well known Minkowski's equation, a quadratic in four coordinates or dimensions.

2. As a first approximation it follows from $c_1 = c\sqrt{1 - \frac{v^2}{c^2}}$ that v can never be greater than c, for in that case the value of c_1 would become imaginary. Hence as a first approximation the velocity c is the maximum attainable.

It has already been seen that as a first approximation the velocity of light is absolute as $v = c$ when v' is put equal to c in $v = \dfrac{v' - u}{1 - \frac{v'u}{c^2}}$.

3. As a second approximation the formula is $\tan \alpha = \dfrac{v}{c}$

and so
$$c_1 = c. \cos \alpha = \frac{c}{\sqrt{1 + \frac{v^2}{c^2}}}$$

$$= c\sqrt{1 - \frac{v^2}{c^2} + \frac{v^4}{c^4} - \text{etc.}}$$

It is also clear that there is nothing absurd in $v > c$; that would merely reduce the value c_1 still more.

If $v = c$, then $c_1 = \dfrac{c}{\sqrt{2}}$. Hence the effective path makes an angle of 45°.

If $v > c$, then the angle α is greater than 45° and tends to 90° as v tends to ∞. Hence any velocity up to ∞ is permissible and is in fact a perfectly legitimate possibility.

4. The physical explanation is that the direction of propagation is shifted forwards and so the effective velocity is diminished to its component $c \cos \alpha = c. \cos \left(\sin^{-1}\frac{v}{D}\right) = c\sqrt{1 - \frac{v^2}{D^2}}$ nearly. The interval ds is nothing but $c_1\,dt$, i.e., $c \cos \alpha.\,dt$, the projection of the distance travelled by light. The anomaly has crept in because of assuming that time is a fourth co-ordinate with the unit $\sqrt{-1}$. By using an artificial fourth dimension, time has been welded into space, and made wholly imaginary, and space itself altogether illusory and fantastic.

Section VI

FRESNEL'S CONVECTION FORMULA

1. From the formula (23·11) for the addition of velocities

$$v = \frac{v' - u}{1 - \frac{v'u}{D^2}},$$

we can easily deduce Fresnel's convection formula by putting c'_1 for v, c_1 for v', c for D and $\frac{c_1}{c} = \frac{1}{\mu}$, where c'_1 and c_1 are the velocities of light in moving and stationary water, and u is the velocity of water.

$$\begin{aligned} \text{Then } c'_1 &= (c_1 - u)\left(1 + \frac{c_1}{c^2}.\,u\right) \text{ nearly} \\ &= (c_1 - u)\left(1 + \frac{1}{\mu}.\frac{u}{c}\right) \\ &= c_1 - u\left(1 - \frac{1}{\mu^2}\right). \end{aligned}$$

The motion of the earth is neglected because it is the same in both the cases.

2. A higher approximation can be deduced from the formula

$$v = \frac{(v' - u)\left(1 + \frac{v'}{D} - \frac{u}{D}\right)}{\left(1 + \frac{v'}{D}\right)\left(1 - \frac{u}{D}\right)}$$

$$\text{as} \quad c'_1 = \frac{(c_1 - u)\left(1 + \frac{c_1}{c} - \frac{u}{c}\right)}{\left(1 + \frac{c_1}{c}\right)\left(1 - \frac{u}{c}\right)} = (c_1 - u).\,\frac{1 + \mu - \frac{u}{c}}{\left(1 + \mu\right)\left(1 - \frac{u}{c}\right)}$$

3. Hoek's and Fizeau's experiments and all other experiments involving an addition of velocities can be explained in a similar way.

SECTION VII

MICHELSON AND MORLEY'S EXPERIMENT

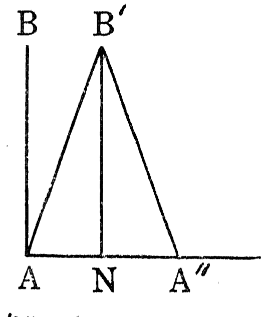
1. If the velocity of light is not independent of its source, then light behaves like a material particle and Michelson's experiment is easily intelligible, as was shown in Chapter II, Section VI, p. 32. The effect is then really null, and it will be absolutely impossible to find out the velocity of the earth by any experiment with light produced on the earth.

If de Sitter's test of the Binary Stars and Majorana's Moving mirrors experiment be right, and the velocity of light is independent of its source, like that of gravitons, as has been assumed in this chapter throughout, then the effect is not null, but of a very small order.

2. It has been wrongly supposed in Relativity that length contracts in the direction of motion and that t_1+t_2, the time taken in the double journey instead of being equal to $\dfrac{l}{c+v}+\dfrac{l}{c-v}=\dfrac{2l.c}{c^2-v^2}$ is $\dfrac{2cl\sqrt{1-\dfrac{v^2}{c^2}}}{c^2-v^2}$.

If length were to contract along the path of light then, there ought to be a corresponding contraction along AB′ and B′A″ as well. These lengths would diminish in the ratio

$$\sqrt{1-\frac{1}{c^2}\left(\frac{v}{c}.v\right)^2}=\sqrt{1-\frac{v^4}{c^4}}.$$



The assumption in Relativity that a right-angled triangle consisting of a solid sheet B′NA can move in the direction of AN with velocity v in such a way that B′N does not change, angle B′NA does not change, the side AN contracts to AN. $\sqrt{1-\dfrac{v^2}{c^2}}$ and yet side AB′ contracts to AB′. $\sqrt{1-\dfrac{v^4}{c^4}}$ while AB′ does not rotate and B′ remains the common point of AB′ and NB,′ is an utterly impossible hypothesis, for there is no reason why AB′ should rotate unsymmetrically round one end B′ only.

3. Really the whole time taken is $t_1+t_2=\frac{2lc}{c^2-v^2}=\frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)$ nearly.

It is also the fact that light along AB′ when received at B′, which is moving with velocity v is shifted forward by an angle $\sin^{-1}\frac{v}{c}$ owing to the principle of aberration as previously explained and behaves as if it came along A_1B' where $AB'A_1 = \alpha$.

If, therefore, the eyepiece remains fixed so as to receive only the light coming along AB′, the component of the velocity of light along AB′ is reduced to $c_1=c.\cos\alpha = c\sqrt{1-\frac{v^2}{c^2}}$. It is this component which is reflected and travels along B′ A″. Hence the time taken is given by $c_1{}^2\,t^2 = l^2 + v^2\,t^2$.

Therefore $2t=\frac{2l}{\sqrt{c_1{}^2-v^2}} = \frac{2l}{\sqrt{c^2-2v^2}} = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)$ nearly

which is exactly the same as before.

As it is only the apparent direction of light that is perpendicular, the factor has been taken as $\sqrt{1-\frac{v^2}{c^2}}$. But $\frac{1}{\sqrt{1+\frac{v^2}{c^2}}}$ would also give practically the same result.

4. The experiment does not really give a null result, but the result is too small and the difference $= \frac{2lc}{c^2-v^2} - \frac{2l}{\sqrt{c^2-2v^2}} = \frac{-l}{c}\cdot\frac{v^4}{c^4}$ nearly, which is not detectable.

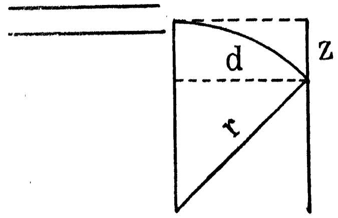
Miller's[21] observation that there is a definite drift of about 7 km. a second does not appear to be correct.

5. In all experiments in which a displacement of the interference fringes is observed, both the rays of light, before they split up, are travelling in a common direction, and after reflections and refractions they both again travel in another common direction. Hence they both experience a rotation, which is exactly the same fraction or multiple of 2π, even though in opposite senses, and so the effect of the motion of the earth is nullified It is, therefore, no wonder that a rotation of the apparatus, which makes both the rays rotate through the same angle, makes no practical difference.

SECTION VIII

BUCHERER'S EXPERIMENT

(See Saha's Modern Physics,[22] pp. 69—71.)



1. The electron with the maximum deflection travels along a circular path where

$$r^2 = (r-x)^2 + a^2$$

and $$\frac{mv^2}{r} = \mathrm{H}.\ e.\ v.$$

Hence $$\frac{e}{m} = \frac{2v}{\mathrm{H}} \cdot \frac{z}{a^2+x^2} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23\cdot2)$$

Experiments show that when $\frac{v}{c}$ changes, the value $\frac{e}{m}$ does not remain constant, unless it is multiplied by a factor nearly equal to $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.

Accordingly the conclusion is drawn that the mass m_0 has changed in the ratio $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, which is supposed to verify the result in Special Relativity approximately.

2. But the magnetic force takes time, say $\frac{1}{\mathrm{D}}$ seconds to act, and so can be represented by a field of flow with the velocity $\mathrm{D}=c$ of electric waves.

The component of the magnetic field parallel to the direction of the motion of the electron does not produce any force whatsoever along that direction; the only effective component is that perpendicular to the path. Further, the maximum deflection is given by the electron making an angle $\frac{\pi}{2}$ with the effective direction.

Now the effect of the principle of aberration, for electrons emerging apparently perpendicular to the field, is to shift the direction of the field forward by an angle $\sin^{-1}\frac{v}{c}$. Accordingly the component of the effective force perpendicular to the path of the electron instead of being $e.\ v.\ \mathrm{H}$. becomes

$$e.\ v.\ \mathrm{H} \quad \cos\left(\sin^{-1}\frac{v}{c}\right) = ev\mathrm{H}.\sqrt{1-\frac{v^2}{c^2}}.$$

Hence $\frac{e}{m} = \frac{2v}{H.\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{x}{a^2+x^2}$, agreeing with the experimental results.

3. It is now obvious that the mass of the electron does not really change with its velocity, but the direction of the field acting upon it is changed, so that the effective component is diminished.

4. For higher approximations, the corrections must include (i) the angle made by the electron with the real direction, (ii) the curved motion of the electron, (iii) its spin, (iv) finiteness of the solenoid, producing a non-uniform field, (v) variability of the current, (vi) the edge effect at the point where the electron leaves the discs, (vii) the ordinary potential of the magnetic field must also be replaced by the vector potential $A=\left[\frac{e.v}{c.r}\right]$ where the square brackets mean that the retarded potential at time $\left(t-\frac{r}{}\right)$ should be taken into account. (See Livens[23] : The Theory of Electricit , p. 504, § 566) These considerations will be applied in a later chapte..

SECTION IX

SOMMERFELD'S FINE STRUCTURES OF SPECTRAL LINES

Niels Bohr assumes that there are several levels of energy, which an electron in its motion can occupy. If the orbit of an electron were not to rotate, then the energy, whether the orbit be circular or elliptical, would be the same, and the fall from one or the other kind of orbit would yield the same difference of energy so long as the major axis is equal to the radius of the circle. Hence the frequency of the emitted radiation from all atoms, no matter in what kind of orbits they are moving, when brought to a focus through a lense, would show one single spectral line. On the other hand, if the elliptical orbits were to rotate and their rotations be dependent on their eccentricities, and circular orbits of course do not rotate, there would be a difference in the energies due to the rotations of the elliptical orbits. Then the differences of energy due to a fall from one elliptical orbit would not be the same as that from another with a different eccentricity, although the major axis be the same. Accordingly the spectral line instead of being a single line would have fine structure. This is actually observed.

Sommerfeld[24] has tried to explain this phenomenon by applying the formula for the increase of mass with velocity in Special Relativity, when an electron goes round in its orbit. The assumption that the

change in Kinetic energy is due to the increase of the longitudinal mass breaks down in the case of the perihelion of Mercury, where the transverse mass also cannot be neglected.

The obvious explanation, however, is that instead of the mass changing from time to time, the force exerted by the nucleus on the electron has different effective components on the electron, depending on its tangential velocity for the time being; the effective direction of the force is shifted forward by the angle $\tan^{-1}\frac{v}{D}$ The problem then becomes identical with that in Chapter I, Section V, p. 8, the mass M of the Sun will correspond to the charge (Ze) on the nucleus, and the mass m of a planet will correspond to the charge e on the electron, and the constant of gravitation will be replaced by unity.

But as the velocity of the electron is now comparable to that of light $\frac{1}{D}\cdot\frac{rd\theta}{dt}$ is not so small a quantity as to justify the neglect of all its higher powers so that $\left(\frac{1}{D}\cdot\frac{rd\theta}{dt}\right)^2$ must now be retained in the equations on page 9 of Chapter I.

The constants of integration will be fixed by modified quantum conditions, with the help of

The Third Universal Principle

When a force travelling with a finite velocity D acts on a thin spherical shell of radius a, spinning with an angular velocity w round an axis through its centre perpendicular to the force, the effect is to decrease or increase its spin according as the force is repulsive or attractive, but always to decrease the component of the force along the diameter by the factor $\sqrt{\frac{1}{1+\frac{a^2w^2}{D^2}}}$.

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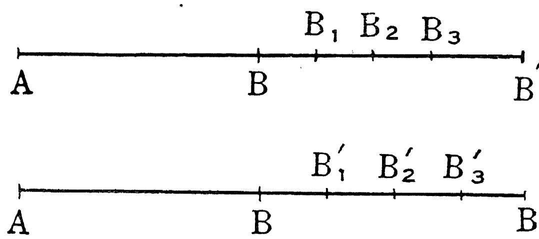
APPENDIX TO CHAPTER I

Motion of two bodies in their line of centres

In paragraph 2 of Section IV in Chapter I (p. 7), it was pointed out that as the gravitons, instead of overtaking B at the distance r, overtake it at distance $r+\delta r$, the intensity of the force is reduced in the ratio $\dfrac{1}{\left(1+\dfrac{\delta r}{r}\right)^2}$

The following is the more detailed Mathematical deduction of the same.

(1) *When only one body is moving.*



Suppose that gravitons leave the Sun A at infinitesimal intervals dt, and overtake the moving planet B at the further *additional* distances δr_0, δr_1, δr_2 etc.

Then for the graviton which leaves A at $t=0$,

$$\delta r_0 = \frac{r}{D-v}\, v \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (24{\cdot}1)$$

So the total distance $AB'' = r + \delta r_0 = r + \dfrac{r}{D-v}\, v = \dfrac{r}{1-\dfrac{v}{D}}$ ... (24·2)

For the graviton which leaves at $t=dt$

$$\delta r_1 = \frac{r+vdt}{D-v}\, v \quad \dots \quad \dots \quad \dots \quad \dots \quad (24{\cdot}3)$$

and therefore $AB'_1 = r+vdt+\delta r_1$.

$$= (r+vdt)+\frac{r+vdt}{D-v}v$$

$$= (r+vdt)\frac{1}{1-\frac{v}{D}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24{\cdot}4)$$

For the graviton which leaves at $t=2dt$

$$\delta r_2 = \frac{r+2\,vdt}{D-v}r$$

and therefore $AB'_2 = r+2vdt+\delta r_2$

$$= (r+2vdt)\frac{1}{1-\frac{v}{D}} \quad \ldots \quad \ldots \quad \ldots \quad (24{\cdot}5)$$

etc., etc., etc.

Newton supposed that successive impulsive pulls at B_1, B_2, B_3 etc. are $\frac{m\mu dt}{r^2}$, $\frac{m\mu dt}{(r+dr)^2}$, $\frac{m\mu dt}{(r+2dr)^2}$, etc.

But the effective impulsive pulls really act at B'_1, B'_2, B'_3 etc. and are $\frac{m\mu dt}{(r+\delta r_0)^2}$, $\frac{m\mu dt}{(r+vdt+\delta r_1)^2}$, $\frac{m\mu dt}{(r+2vdt+\delta r_2)^2}$ etc.,

$$i.e., \quad \frac{m\mu dt\left(1-\frac{v}{D}\right)^2}{r^2}, \quad \frac{m\mu dt\left(1-\frac{v}{D}\right)^2}{(r+vdt)^2}, \quad \frac{m\mu dt\left(1-\frac{v}{D}\right)^2}{(r+2vdt)^2}, \text{ etc}$$

$$\text{or} \quad \frac{m\mu dt\left(1-\frac{v}{D}\right)^2}{r^2}, \quad \frac{m\mu dt\left(1-\frac{v}{D}\right)^2}{(r+dr)^2}, \quad \frac{m\mu dt\left(1-\frac{v}{D}\right)^2}{(r+2dr)^2} \text{ etc.} \quad (24{\cdot}6)$$

But vdt is an infinitesimal quantity, while δr is finite; also $n.vdt$ is finite, if a small distance be considered. Hence the displacement decreases all the effective impulses in the ratio $\left(1-\frac{v}{D}\right)^2$.

But the decrease in frequency due to the Doppler effect is

$$= \frac{1}{1+\frac{\delta r}{r}} = 1-\frac{v}{D}$$

Hence the total decreased ratio $= \left(1-\frac{v}{D}\right)^3 \quad \ldots \quad \ldots \quad \ldots \quad (24{\cdot}7)$

(2) *When both the bodies are moving.*

Let A and B move in the same direction with velocities u and v'.

Then on the Döppler principle, the frequency of the gravitons is changed in the ratio

$$\frac{1-\frac{v'}{D}}{1-\frac{u}{D}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24{\cdot}8)$$

Hence instead of the Newtonian law of gravitation $-F=G\,\frac{Mm}{r^2}$

the effective law is nearly—$F=G.\frac{Mm}{r^2} \times \frac{1-\frac{v'}{D}}{1-\frac{u}{D}} \times \left\{1-\frac{v'-u}{D}\right\}^2 \ldots(24{\cdot}9)$

$$=\left(1-\frac{v'}{D}\right)\left(1+\frac{u}{D}\right)\left(1-\frac{v'-u}{D}\right)^2 \quad \ldots \quad \ldots \quad (24{\cdot}10)$$

$$=1-\frac{3v}{D} \text{ nearly} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24{\cdot}14)$$

where v is the relative velocity between the two bodies.

This embodies the **Fourth Universal Principle**, already applied in Chapter I Sections IV and V, on the sole assumption that, irrespective of any physical theory of gravitation, whether of emission or absorption of gravitons, the influence of gravitation is propagated outwards with a finite velocity.

NOTE.—While the last proofs are being corrected, Prof. M. N. Saha, D. Sc., F.R.S., to whose encouragement I owe much, has kindly drawn my attention to a recent paper of P. Jordan (*Zeitschrift fur Physik*, **93**, 464, 1935) in which he alludes to the possibility of the existence of gravitations quant. A physical theory of gravitons was published by me in September 1933, and a mathematical theory in July 1934.

REMARKS BY PROF. A. C. BANERJI

Allahabad, January 31, 1935.

I have great pleasure in congratulating Sir Shah Muhammad Sulaiman for his accuracy in the mathematical working out of his theory in which I could not find any flaw. If any criticism is to be made, it can only be levelled against the assumptions he has made. The champions of the theory of Relativity emphasise the fact that it is one comprehensive theory which has been able to explain several phenomena that cannot be explained by Newtonian Mechanics. We have yet to see whether Sir

Shah Muhammad's theory has got this comprehensive nature or whether it is a set of disconnected assumptions giving separately individual results which may otherwise be deduced from the theory of Relativity. Moreover, if his theory is to replace the theory of Relativity, his assumptions should be as few, if not fewer, and simpler than those made in the theory of Relativity. Further, from his theory it should be possible to obtain all those results got already by Relativity, and, if possible, his theory should be able to explain some other phenomena which cannot be explained by Relativity.

If I understand rightly, Sir Shah Muhammad's assumptions are so far the following:—

(*i*) Gravitational effect and all other physical effects (electric and magnetic) are propagated with a finite velocity.

(*ii*) The law of gravitational attraction between two bodies relatively at rest is different from the law of attraction between the same two bodies when in motion relatively to each other.

(*iii*) The theory of self-accelleration as deduced from the assumption that the velocity of gravitons or light corpuscles in space is independent of that of its source.

(*iv*) The theory of self-acceleration.

In my opinion these ideas are at least as extraordinary as those in Relativity. Moreover, his theory of self-acceleration seems to me to be quite extraordinary. He takes the case of a spherical body moving with a velocity V along a straight line. He assumes that at least tiny light particles are radiating normally with velocity D which is about the velocity of light. He further assumes that dynamically the resultant effect of every particle in a sphere emanating corpuscles is practically the same as if the whole emission of corpuscles were from the entire mass concentrated at the centre and hence the motion is wholly radial. As his gravitons are the cause of gravitation they are not material particles subject to gravitation. He takes the absolute velocity of any such particle when it emerges to be wholly radial. It has momentum μD in space and had momentum $\mu V \cos\theta$ at the centre before emerging and so it takes away momentum for $\mu D - \mu V \cos\theta$ from the body as it emerges. These are *new* assumptions combining Newton's and Einstein's conceptions; and they contravene classical Mechanics in as much as cross-radial momentum is not taken account of. It is an uncommon assumption for a material particle. It is yet to be seen how his theory of self-acceleration would work out in the case of unsymmetrical bodies and bodies of irregular shape.

Validity of his rotational theory of light can only be judged when it is mathematically worked out. His formula for the composition of velocities is very interesting and is based on the approximate absoluteness of the velocity in a *to and fro* journey. He has tried to deduce physical explanations of the factor in Lorentz transformations from his assumption that all effects take time to act. For velocities which are small compared with the velocity of light the analogous formula in Relativity becomes a second approximation to his formula. α and β particles have velocities comparable to that of light, and by performing experiments with these particles it is for the physicists to test which of the two formulæ will give results in accordance with observation,

THE ABSORPTION SPECTRA OF THE VAPOURS OF SULPHUR MONOCHLORIDE AND THIONYL CHLORIDE AND THEIR CONSTITUTIONS

By Hrishikesha Trivedi

Physics Department, Allahabad University

Communicated by Prof. M. N. Saha

Received December 13, 1934

It is well known that chemical constitution of compounds containing one or more atoms with multiple and variable valency can be represented by a number of different formulæ. An example is afforded by sulphur monochloride for which two constitutional formulæ have been proposed, *viz.*, $Cl-S-S-Cl$ and $S=S=Cl_2$. There is no such ambiguity in the case of analogous substance obtained by the substitution of oxygen for one of the sulphur atoms, *viz.*, thionyl chloride for which the formula $O=S=Cl_2$ is accepted by everybody. The reasons for this divergence of view will be apparent from the following quotation from Mellor's *Comprehensive Treatise on Theoretical and Inorganic Chemistry*, Vol. X, page 642. "Its constitutional formula may be $Cl-S-S-Cl$ or according to A. Michaelis and O. Schifferdecker, H. L. Olin, L. Carius and T. E. Thorpe, $S=S=Cl_2$, analogous with thionyl chloride $O=S=Cl_2$, and it has accordingly been called sulphothionyl chloride. B. Holmberg said that its action on mercaptan favours the formula $Cl-S-S-Cl$. According to G. Bruni and M. Amadori, just as a series of persulphides is produced by the introduction of sulphur to hydrogen sulphide $H-S-H$, $H-S_2-H$ and $H-S_n-H$, so may the corresponding sulphur chlorides, sulphur dichloride $Cl-S-Cl$, sulphur monochloride $Cl-S_2-Cl$ and $Cl-S_n-Cl$ be possible."

I have tried to solve this problem by studying the absorption spectrum of the vapours of sulphur monochloride and thionyl chloride. Very little work of this nature has previously been done on these compounds. Lowry and Jessop[1] found that sulphur monochloride is transparent to light of wavelengths 5200Å and 5400Å. They found that it has a strong maximum absorption in the ultraviolet, log ε being 3·8 at 2660Å; but it

could not be estimated photometrically on account of the absorption of ultraviolet light by the dichloride. In the present case the experiment was done in such a way that no trouble could arise by the presence of dichloride as was the case in Lowry and Jessop's experiment.

Experimental Procedure

Both the substances are liquids. Their properties are given below:

Sulphur monochloride	Thionyl chloride
A yellowish red heavy liquid, fumes in air; its boiling point is 138°C. Its vapour pressure in mms. of mercury at various temperatures is 0°C 3·7; 10°C 6·4; 20°C 10·7; 40°C 28·0	A colourless liquid boiling at 80°C; when heated it dissociates into SO_2, sulphur monochloride and chlorine.

Both of the substances taken were Merck's extrapure samples. The absorption chamber was a pyrex glass tube one centimetre in diameter having a length of two metres. The two ends of the tube were closed with quartz windows which were fixed on to the ground ends of the tube by means of liquid sodium silicate. The liquid under investigation was kept in a bulb which was connected through a stopcock to the absorption chamber by means of a side tube. The absorption tube was connected by means of a side tube and stopcock to a pump which was kept running continuously. The pressure of the vapour inside was measured by means of a manometer. The continuous running of the pump served the purpose of maintaining a constant current of the vapours passing through the absorption vessel, the rate being regulated by means of the stopcocks. The vapour within the absorption chamber was kept successively at the various values (<0·1, 0·1, 2, 5, 7, 10 mms. of Hg). This circulation ensured that all the products of any dissociation that may have taken place (there was very little possibility that any dissociation had taken place, as the temperature of the vessel was not greater than 17°C; at this temperature the dissociation is negligible for either of the two substances) in the absorption chamber will be drawn out and will not vitiate the experimental results.

For the source of continuous radiation a hydrogen tube run by a 2 K. W. transformer was used. Photographs were taken with a quartz E_3 spectrograph. Ilford special rapid plates sensitised by means of Nujol

paraffin were used for photographing the spectra. This treatment made the plates sensitive right up to 1850 A.U. A copper arc was used as a standard.

Results and Discussion

The results of this investigation are as follows:

1. The absorption spectrum of sulphur monochloride shows two regions of continuous absorption when the pressure of the vapour in the absorption chamber was 0·1 mm. of mercury or even less. In between there was a patch of retransmitted light. As the pressure of the absorbing vapour increased the intensity of the retransmitted light gradually diminished till at a pressure of 2 mms. there was no retransmitted light and the two regions of continuous absorption merged into one. As the pressure of the vapour increased the long wavelength limit of the regions of absorption went on receding towards the red, till it adopted a stationary position at a pressure of 5 mms. With further increase of the pressure of the absorbing vapour there was no change in the position of the long wavelength beginning. This shift with increasing pressure of the vapour was found even when the two regions of absorption had not merged into one, and the difference between their long wavelength beginnings was practically the same throughout that range of pressure for which the two regions were distinctly separate.

The long wavelength beginnings of the two regions of absorption were 2740Å = 104·3 Kcals and 2135Å = 134 Kcals respectively with the vapour at about 0·1 mm. pressure. When the two regions merged into one, the long wavelength beginning of the absorption of the vapour at 5 mms. or more is 4290Å = 66·6 Kcals.

All the cuts are fairly sharp. The beginning of absorption was determined from the microphotograms of the spectra. In this connection the author would like to thank Prof. Ashutosh Mukerji of Patna for having kindly allowed the use of the microphotometer belonging to Science College, Patna.

2. In the case of thionyl chloride, too, there were two regions of continuous absorption separated by a retransmitted patch of light. The absorption spectrum was photographed when the pressure of the vapour inside the chamber was very small ($<$ 0·1 mm.). In one of the photographs certain absorption bands appeared along with the continuous absorption. As the pressure of the vapour was increased the two regions of continuous absorption pushed further towards the red end of the plate, till it became stationary at a pressure of 7 mms. The difference between

the beginnings of the two regions was roughly the same as they shifted towards red with increasing pressure.

The bands were extremely faint and their heads were not clearly developed. Their measurement was, therefore, extremely difficult and there is, consequently, a good deal of uncertainty in locating their positions. The various heads were at λλ 2107, 2099, 2086, 2076, 2067, 2047. These bands could be identified as those of CO. It must have come up as an impurity from the grease applied to the stopcock. The long wavelength beginnings of the two regions of continuous absorption were λλ 2980 and 2040 when the pressure of the gas was less than 0·1 mm. of Hg. When the pressure was 7 mms. the long wavelength beginning of the region of continuous absorption was 3215Å.

Coming to the interpretation of these results we see that sulphur monochloride is, according to the majority of the chemists, represented by either of the two constitutional formulæ

$$(a)\ S{=}S\begin{matrix}\diagup Cl\\ \diagdown Cl\end{matrix}, \qquad (b)\ Cl{-}S{-}S{-}Cl.$$

In (*a*) one of the sulphur atoms is taken to be divalent and the other as quadrivalent. Let us first consider the optical dissociation of $S{=}S\begin{matrix}\diagup Cl\\ \diagdown Cl.\end{matrix}$ The light dissociates the molecule into either S and SCl_2 or S_2Cl and Cl. In the first alternative, the two regions of continuous absorption correspond to dissociation into SCl_2 and S in its ^3P and ^1D states respectively. In the second alternative the two regions are due to the two different electronic states of SCl_2 molecule as the ^2P states of chlorine are too near to one another to give two different regions of continuous absorption. The first alternative can be tested. The second one cannot, however, be tested for want of data.

We can calculate thermochemically the energy of dissociation of S_2Cl_2 into S and SCl_2 as follows :

(1)[2] $2\,[S]_{rhombic}+Cl_2=S_2Cl_{2_{liq}}+14{\cdot}5$ Kcals

(2)[3] $S_2Cl_{2_{liq}}+Cl_{2_{gas}}=2\,SCl_{2_{liq}}+9{\cdot}8$ Kcals

(3)[4] $S_2Cl_{2_{liq}}+8{\cdot}5$ Kcals $=S_2Cl_{2_{gas}}$

(4)[5] $2\,[S]_{rhombic}+29{\cdot}3$ Kcals $=S_{2_{gas}}$

(5)[6] $S_{2_{gas}}+102{\cdot}6$ Kcals $=2\,S_{gas}$

(6)[7] $SCl_{2_{liq}}+4$ Kcals $=SCl_{2_{gas}}$

We can very easily calculate from the above the energy required to dissociate S_2Cl_2 into S and SCl_2. It amounts to 63·8 Kcals. The

long wavelength beginning of the region of continuous absorption, which has been taken to give the energy of dissociation of the molecule for many compounds by various workers, amounts to 66·6 Kcals. The values are in very nice agreement. This shows our assumption, that S_2Cl_2 dissociates into S and SCl_2, to be valid. This is further substantiated by the presence of two regions of continuous absorption at very low pressures of the vapour. The difference between their beginnings amounts to 134−104·3=29·7 Kcals. The value of $^3P-^1D$ for sulphur has been found to be about 26 Kcals[8]. This agreement is very nice and confirms our view.

If the molecule S_2Cl_2 breaks up optically into S and SCl_2, its constitution cannot be represented by any other formula but the following

$$S{=}S\left\langle\begin{array}{l}Cl\\Cl\end{array}\right.$$

and it does not dissociate optically in any other way but the abovementioned

Coming to $SOCl_2$ we find that it behaves in an exactly analogous manner. Unfortunately we cannot calculate, for want of data, the energy required to dissociate the molecule into O and SCl_2 or S and OCl_2, depending upon whether the molecule is $O=S\left\langle\begin{array}{l}Cl\\Cl\end{array}\right.$ or $S{=}O\left\langle\begin{array}{l}Cl\\Cl,\end{array}\right.$ thermochemically. The only data we know is that the heat of formation of liquid thionyl chloride is 47·2 Kcals per mol and its heat of vaporisation is 54·45 cals per gram. These are insufficient to calculate the required energy. We rely on the interpretation of the two regions of absorption and to infer therefrom the constitution of the molecule. The difference between the long wavelength beginning of the two regions of continuous absorption is 140 −96 = 44 Kcals. This agrees well with the difference between the energies of ^3P and ^1D states of oxygen, which is equal to 1·9 volts[9]. This shows that the molecule of thionyl chloride has the structure

$$O{=}S\left\langle\begin{array}{l}Cl\\Cl\end{array}\right.$$

and dissociates into O (^3P, ^1D) and SCl_2. Fortunately the chemists do not advance any other alternative formula for this molecule and the spectroscopic observations substantiate their observations regarding the constitution of thionyl chloride. In the case of sulphur monochloride,

however, the spectroscopic result confirms the view that the constitution can be represented as

$$S = S \begin{cases} Cl \\ Cl \end{cases} .$$

It further shows that the binding between S and S in the monochloride and between O and S in the thionyl chloride is electronic, a view which has already been put forward by Prideaux[10] for the latter case.

Summary

Absorption spectra of sulphur monochloride and thionyl chloride was studied. Both the substances show, at low pressures, two regions of continuous absorption separated by a region of retransmission. Their long wavelength beginnings are at 2740Å and 2135Å in the case of S_2Cl_2, whereas for $SOCl_2$, they were at 2980Å and 2040Å. The long wavelength beginning shifts towards red with increasing pressure. The difference between the energies corresponding to these beginnings is equal to that of $^3P-^1D$ of S in the case of S_2Cl_2 and $^3P-^1D$ of O in the case of $SOCl_2$. It is shown that the constitution of these molecules is as

$S{=}S \begin{cases} Cl \\ Cl \end{cases}$ and $O{=}S \begin{cases} Cl \\ Cl \end{cases}$ and the binding is electronic.

Acknowledgement

I gratefully thank Prof. M. N. Saha, F.R.S., for the encouraging kindness he bestowed on me.

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STUDIES ON THE FAMILY HETEROPHYIDÆ ODHNER, 1914.

Part I.—On a New Distome from the Indian Fishing Eagle—*Haliaeetus leucoryphus*—with Remarks on the Genera *Ascocotyle* Looss, 1899, and *Phagicola* Faust, 1920.

By Har Dayal Srivastava

Zoology Department, University of Allahabad

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The family Heterophyidæ is composed of a number of genera of small flukes which are usually parasitic in fish-eating Vertebrates. The members of this family were known to occur in only birds and mammals till 1932 when Mueller and Van Cleave enlarged the scope of the family to include certain genera which are parasitic in fishes. The family is of considerable interest in human and veterinary medicine. Because of their occurrence in birds and mammals and their normal or potential infestation of man, the Heterophyids in other countries have received serious consideration. From India, however, this is the first record of the occurrence of a member of this family.

Ascocotyle Looss, 1899.

Looss in 1899 created the genus *Ascocotyle* for *Distomum coleostomum* Looss, 1896, and added one more new species to it—*A. minuta*. Subsequently a number of species have been added to the genus, namely: *A. italica* Alessandrini, 1906, *A. angrense* Travassos, 1916, *A. longa* Ransom, 1920, *A. nana* Ransom, 1920, *A. diminuta* Stunkard and Haviland, 1924, *A. angeloi* Travassos, 1928, *A felippei* Travassos, 1928, *A. ascolonga* Witenberg, 1928, *A. arnaldoi* Travassos, 1928, *A. megalocephala* Price, 1932

and *A. puertoricensis* Price, 1932. Faust in 1920 described, from the intestine of the monkey-eating eagle, *Phagicola pithecophagicola* N. Gen., N. Sp. "a fluke which on a restudy," by Faust in 1926 "has been found to belong to the genus *Ascocotyle* and should, therefore, be designated as *A. pithecophagicola.*" In the original description of *P. pithecophagicola* Faust failed to notice the posterior oral appendage and gonotyls and the apparent absence of these structures necessitated the creation of a new genus. Stunkard and Haviland in 1924 split up the genus *Ascocotyle* into two sub-genera: (*Ascocotyle*) with *A. coleostomum* as type and (*Parascocotyle*) with *A. minuta* as type. In his revision of the family Heterophyidæ Witenberg in 1929 recognised the sub-genus (*Parascocotyle*) Stunkard and Haviland as of generic rank and provisionally included in it *Phagicola pithecophagicola.* He further considered *Parascocotyle diminuta* Stunkard and Haviland as a synonym of *P. minuta* Looss, attributing the specific differences between the two species to age and fixation. A year later after examination of the type specimens of *P. pithecophagicola* Witenberg was unable to add anything to the original description of Faust. He, however, suggested that only a redescription of a new material of *Phagicola pithecophagicola* could settle the question whether *Parascocotyle* is synonymous with *Phagicola* or they both are valid genera. Travassos in 1930 and Price, 1933, accepted *A.* (*Parascocotyle*) *dimunita* and *A.* (*Parascocotyle*) *minuta* as valid species Recently Price (1933) has restudied the type specimens of *P. pithecophagicola* Faust and has demonstrated clearly the presence of a long posterior oral appendage, gonotyls and a globular receptaculum seminis, necessitating the identity of the genera *Phagicola* and *Parascocotyle.* Nevertheless Price is still "of the opinion that sufficient differences exist between *Ascocotyle* and *Phagicola* to warrant the latter being considered as a distinct genus." *Ascocotyle plana* Linton, 1928, which Witenberg in 1928 regarded as a synonym of *Pygidiopsis genata* Looss and which Travassos in 1930 regarded as a synonym of *Ascocotyle* (*Phagicola*) *angrense* Travassos, has been recognised by Price in 1933 to be a species of *Pygidiopsis*: *P. plana.*

Ascocotyle (Phagicola) intermedius N. Sp.

A large number of these minute distomes were obtained from the intestine of the Indian Fishing-eagle—*Haliaeetus leucoryphus.* In the

living condition they are quite active and show considerable power of contraction and expansion especially in the anterior half of the body. In their natural habitat the parasites appear like tiny masses of yellowish brown pigment. Normally the body is pyriform in outline but in extended condition the sides become almost parallel while the contracted worm may be as wide as long. The dorsal lip may be extended anteriorly in the form of a triangular process or retracted into a short knob. In permanent mounts the parasite has a thin flat body with a flask-shaped outline, measuring 0·6—0·9 * in length and 0·2—0·38 in maximum breadth across the anterior margin of the ovary. The body is uniformly studded with minute backwardly directed spines of 0·005 × 0·002 size, which diminish both in number and size as they approach the posterior end.

The oral sucker is terminal, measuring 0·04—0·05 in diameter; it is surrounded by two crowns of alternating cylindrical and abruptly-pointed spines, about 28—30 in number. These spines are quickly shed when the worms are placed in normal salt solution or tap water. The spines in the anterior crown measure 0·01—0·013 × 0·003 in size while those in the hinder crown measure 0·009—0·01 × 0·003. The feebly muscular acetabulum, spherical in outline and 0·066—0·077 in diameter, is situated about the middle of the body. The size ratio between the oral and the ventral suckers is as 2 : 3. Both the acetabulum and the genital pore lie in a shallow depression—the ventro-genital sinus—on the ventral body surface. The genital sinus or ductus hermaphroditicus opens in this depression just in front of the acetabulum. The genital opening is guarded on the anterior and posterior sides by two muscular, transversely elongated pads, lenticular in shape,—the gonotyls of Witenberg,—which measure 0·04—0·05 × 0·013 in size.

The excretory pore is terminal lying at the extreme posterior end of the body. The bladder is typical of the genus and consists of a short main stem which bifurcates just behind the receptaculum seminis into two short but wide cornua.

The oral sucker has a posterior hollow, conical prolongation—the oral appendage or caecum—of 0·04—0·05 × 0·03 size, which is situated on the dorsal side of the prepharynx. The prepharynx is long, measuring 0·06—0·09 in length. The muscular pharynx, 0·03—0·04 × 0·02—0·03 in size, is followed by a short oesophagus of 0·03—0·08 length. The wide intestinal caeca are moderately long terminating posteriorly in level with the anterior margin of the ovary.

* All measurements are in m.m.

The gonads are well developed and lie in the posterior half of the body. The testes are situated symmetrically with their long axes directed obliquely, one on each side of the hinder end of the body. They have slightly irregular margins and measure 0·12—0·17×0·07—0·12 in size. The vasa efferentia pass forwards and open into the posterior end of the vesicula seminalis which is enormously developed measuring 0·38—0·42×0·07 in size. It is roughly retort-shaped with its long axis placed transversely in the space between the acetabulum and the receptaculum seminis and is narrowed anteriorly to form a fairly long tubular ejaculatory duct of 0·1—0·12 × 0·03 size which lies to the right side of the acetabulum. Terminally the ejaculatory duct joins the uterus just before the genital pore forming the genital sinus. There is no cirrus sac present.

The ovary is situated a little to the right side about the middle of the post-acetabular region between the right testis and the coils of the uterus. It has an irregular outline, measuring 0·08—0·1×0·11—0·14 in size. The receptaculum seminis, somewhat rounded in outline, lies, in the median line in level with and partially overlapping the ovary and the yolk reservoir. Its size varies with the amount of its contents ranging from 0·08—0·13 in length and 0·1—0·13 in breadth. A short but fairly wide Laurer's canal is present.

The vitellaria consisting of small irregular follicles of 0·01—0·04× 0·008—0·02 size, are profusely developed and extend laterally from the anterior level of the pharynx to the posterior ends of the testes. Anteriorly in the region from the pharynx to a little distance behind the intestinal bifurcation the vitelline follicles of the two sides meet in the median line. A small yolk reservoir lies slightly to the left of the median line between the posterior end of the vesicula seminalis and the receptaculum seminis, partly overlapping the latter.

The uterus composed of a wide S-shaped ascending coil is confined to the space between the testes and the ventro-genital sinus, never extending beyond the latter. It is packed with a fairly large number of large sized, yellowish brown, operculate eggs of 0·03—0·035×0.015—0·017 size.

A. intermedius N. Sp. is assigned to the sub-genus (*Phagicola*) Travassos on account of the length of the oesophagus and of the intestinal caeca and the extent of the uterus. This species resembles the sub-genus (*Ascocotyle*) Travassos in the arrangement of oral spines and the relatively large extent of the vitellaria but it differs from it in the

presence of a fairly long oesophagus followed by long caeca which extend far behind the acetabulum and in the extent of the uterus which never extends in front of the genital sinus; features in which it resembles *A.* (*Phagicola*). It differs from all the species of the latter sub-genus in the enormous development and extent of the vitellaria and in having a double crown of oral spines. Within the sub-genus, in the arrangement of the oral spines it resembles *A.* (*Phagicola*) *angeloi* and *A.* (*Phagicola*) *nana*, in the latter species only the dorsal spines are in double row. *A. intermedius*, however, differs from all the species of the genus in the number of oral spines, much larger extent of the vitellaria and the size of its eggs which are the largest in the genus.

The genus *Phagicola* as now constituted by Price differs from *Ascocotyle* only in the presence of an oesophagus, the length of the intestinal caeca which extend posteriorly beyond the acetabulum, the post-acetabular position of the vitellaria, the extent of the uterus which never extends anteriorly beyond the ventro-genital sinus. The intermediate species described in this paper connects the two genera—*Ascocotyle* and *Phagicola*—as regards the extent of the vitellaria. The remaining important differences between the two genera are the extent of the intestinal caeca and the uterus. The extent of the intestinal caeca cannot in this case be considered of generic importance as all the gradations in their length exist between such forms as *Phagicola minuta* and *P. arnaldoi*. The extent of the uterus alone does not offer a sufficient justification for maintaining two distinct genera. We, therefore, agree with Travassos in reducing the genus *Phagicola* to the rank of a sub-genus.

The diagnosis of the genus *Ascocotyle* as given by Travassos needs certain modifications in the light of the new forms described subsequently. The emended diagnosis of the genus is as follows :—

Minute distomes, body thickly spinose; oral sucker armed with a single or double crown of straight cylindrical spines. Oral sucker continued posteriorly into a distinct appendage; prepharynx long, pharynx well developed and muscular, oesophagus present or absent, intestinal caeca long or short. Acetabulum median, situated in association with the genital sinus in a depression of the ventral body surface. Testes situated one on each side at the hinder end of body; vesicula seminalis and ejaculatory duct well developed. Cirrus sac is absent. Ovary median or slightly to one side, pretesticular; receptaculum seminis large, situated in level with ovary or behind it. Vitellaria lateral, usually post-acetabular sometimes extending as far forward as the pharynx

and meeting mesially near the intestinal bifurcation. Uterus usually post-acetabular, rarely extending as far forwards as the pharynx; eggs large, operculate, measuring 0·015—0·035 × 0·008 – 0·017 in size.

Parasitic in birds and mammals.

Key to the Sub-genera of Ascocotyle Looss

1.	Vitellaria extending in front of acetabulum; Uterus extending in front of ventro-genital sinus; Oesophagus almost absent ...	*Ascocotyle* (*Ascocotyle*)
2.	Vitellaria post-acetabular, except in *A. intermedius*; Uterus confined behind ventro-genital sinus; Oesophagus well developed	*Ascocotyle* (*Phagicola*)

Key to the Species of the Sub-genus Ascocotyle (Ascocotyle)

1.	Vitellaria extending from the level of pharynx to centre of acetabulum	*A.* (*Ascocotyle*) *megalocephala*
	Vitellaria confined between posterior ends of caeca and body	2
2.	Vitellaria pretesticular	*A.* (*Ascocotyle*) *coleostomum*
	Vitellaria extending into testicular region ..	3
3.	Oral spines 36 in number	*A.* (*Ascocotyle*) *felippei*
	Oral spines 32 in number	*A.* (*Ascocotyle*) *puertoricensis*

Key to the Species of the Sub-genus Ascocotyle (Phagicola)

1.	Vitellaria extending from the hinder end up to the level of pharynx	*A.* (*Phagicola*) *intermedius*
	Vitellaria post-acetabular	2

2. Oral spines in double row	3	
Oral spines in single row	4	
3. Oral spines in double row on the dorsal side and in single row on the ventral ...		*A. (Phagicola) nana*
Oral spines in double row on both the surfaces		*A. (Phagicola) angeloi*
4. Genital pore situated at intestinal bifurcation		*A. (Phagicola) pithecophagicola*
Genital pore situated behind intestinal bifurcation	5	
5. Intestinal caeca not reaching ovary ...	6	
Intestinal caeca reaching or extending beyond ovary	8	
6. Oral sucker distinctly larger than acetabulum		*A. (Phagicola) angrense.*
Suckers about equal in size	7	
7. Oral spines 16 in number		*A. (Phagicola) dimunita*
Oral spines 19 (rarely 20 or 18) in number ...		*A. (Phagicola) minuta*
8. Vitellaria follicular	9	
Vitellaria composed of compact masses ...	10	
9. Vitellaria composed of 2—8 follicles on each side—Eggs 0·016—0·018 × 0·009—0·01 in size		*A. (Phagicola) longa.*
Vitellaria composed of 9—12 follicles on each side—Eggs 0·02—0·024 × 0·01—0·012 in size		*A. (Phagicola) arnaldoi*
10. Vitellaria lateral and post-ovarian; Oral appendage and prepharynx equal in length		*A. (Phagicola) ascolonga*

Vitellaria lateral extending up to or beyond ovary ; Oral Appendage half the length of prepharynx ... ... *A. (Phagicola) italica*

This work was carried on under the guidance of Dr. H. R. Mehra to whom I am greatly indebted for his valuable help and advice. I am also obliged to Dr. D. R. Bhattacharya for laboratory facilities in the Department. I am grateful to the Lady Tata Memorial Trust, Bombay, for the grant of a research scholarship for investigations in Helminthology.

Explanation of Plate and Lettering

Plate No. I. Ventral view of *Ascocotyle (Phagicola) intermedius* N. Sp.

(The oral spines are not true to the magnification scale.)

Act.	Acetabulum.		
D. ej.	Ductus ejaculatorius.	Ph.	Pharynx.
D.h.	Ductus hermaphrodificus. or genital sinus.	Pp.	Prepharynx.
Gt.	Gonotyl.	R.s.	Receptaculum seminis.
I.c.	Intestinal caecum.	T.	Testis.
O.a.	Oral appendage.	Ut.	Uterus.
Oes.	Oesophagus.	Vit.	Vitellaria.
O.s.	Oral sucker.	Y.r.	Yolk reservoir.
O.sp.	Oral spines.	V.s.	Vesicula seminis.
Ov.	Ovary.	V.g.s.	Ventro genital sinus.

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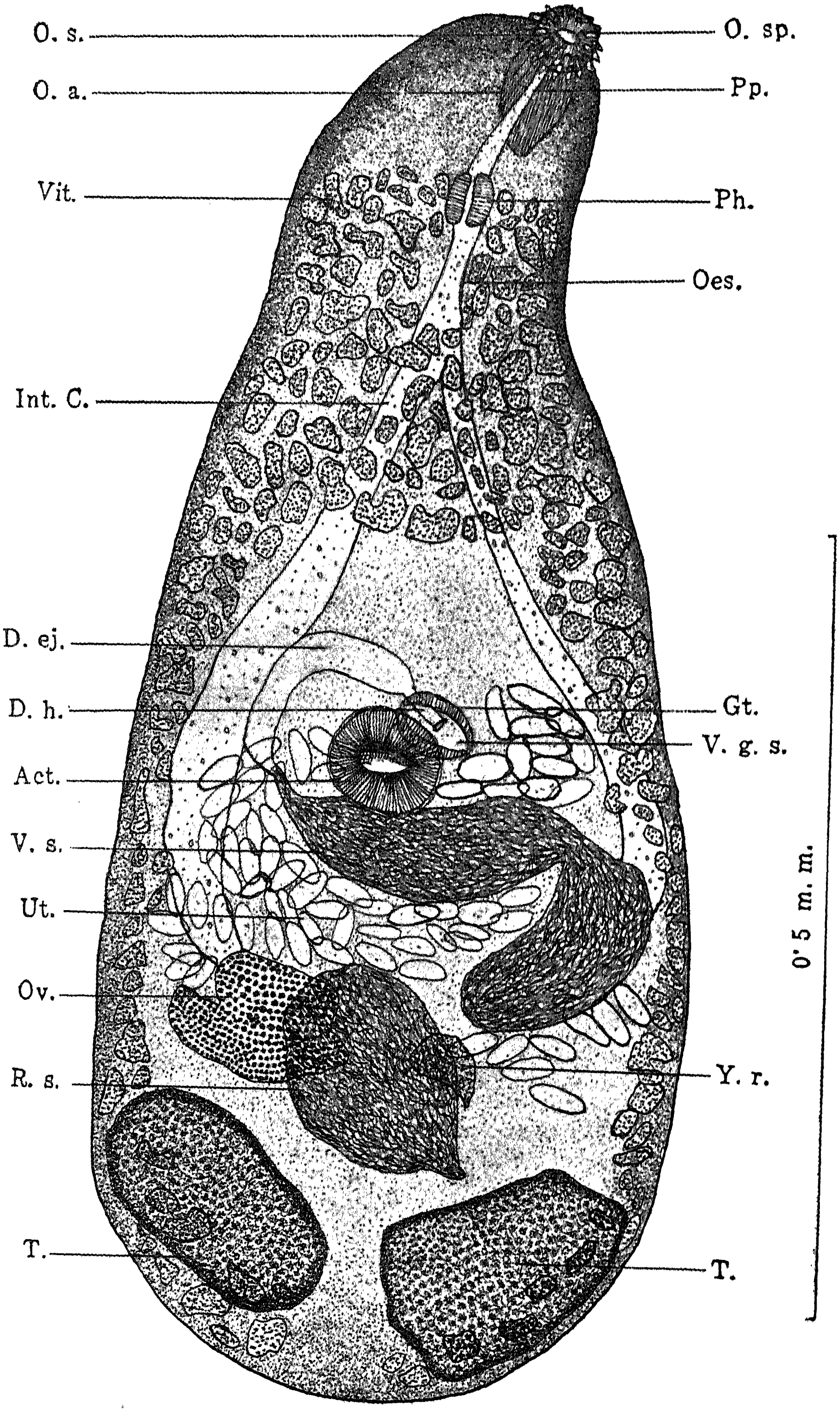
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NOTES ON A CASE OF UNILATERAL ATROPHY OF TESTIS IN THE COMMON WALL GECKO (*HEMIDACTYLUS FLAVIVIRIDIS* RUPPEL).

By S. K. Dutta

Zoology Department, University of Allahabad

Received August 22, 1934

While examining the gonads of a number of wall-lizards (*Hemidactylus flaviviridis* Ruppel) a specimen was found with but one testis of normal size on the left side. On careful dissection the other testis—the one on the right side—was found to consist of extremely attenuated and degenerate seminiferous tubules of microscopical size attached to the Vas deferens. The justification for describing this abnormal feature arose from the fact that on looking up the literature it was found that no case of testis deformity in lizards was on record.

The abnormality of testis has been studied by Fell (4) in a ram; by Crew and Fell (3) in a goat, a cat, a rabbit, a frog, and a human being; by Kennedy (6) in a rat; and by Bhattacharya and Das (1) in a frog. Interesting accounts of cases of hermaphroditism and diseases of the reproductive organs in animals have been recorded by Tichomrow (9) in birds and mammals, Sutton (8) in frogs, birds and mammals, and Bhaduri (2) in *Rana tigrina*. Very unusually, sixteen cases of pigeons without a discernible gonidial tissue have been found and described by Oscar Riddle (7). The absence of gonads was complete and permanent; not temporary or recent. The gonadless condition is regarded as purely developmental. In spite of the complete absence of testicular tissue the birds showed masculine behaviour.

Thus from the literature cited above it will be seen that there is no account extant of the abnormal structure of the lizard testis and I thereforet ake the opportunity of recording the case of atrophy of a testis in a lizard and describing the degenerate cellular structure of the organ.

The entire gonad was dissected out and fixed in Champy's chrome-osmic fixative. Sections were cut by paraffin method and stained in iron haematoxylin.

The Healthy Testis

The left testis showed a normal structure in every respect. The tunic investing the testis was healthy, the seminiferous tubules were large and spermatozoa were present in the lumen of each tubule. Stages of spermatozoa could be seen in sections and the organ appeared to be in active condition. The inter-tubular tissue was composed of connective tissue cells; the interstitial cells could not be distinguished. The epididymis and the vas deferens were in every way typical.

The Degenerate Testis

The histological study of the atrophic testis of the right side on the other hand presents several points of interest (Fig. 2). In the first place the fibrous tissue beneath the investing membrane of the testis (Tunica Albugenia) has developed to a great extent in several places. The outline of the seminiferous tubules is very ill defined and the degeneration of the germinal epithelium is considerable. The lumina of the degenerate tubules were all filled with a coagulated fluid. The formation of such fluid has been observed and described by Crew and Fell (3) in the displaced and undescended testis of the goat, cat and rabbit. They have identified spherical colloid bodies in large numbers. The coagulated mass in the centre of the tubules is regarded as a degenerate product of the germinal epithelium.

The reduced size of the right testis is evidently due to its wasting away for lack of nourishment. The epididymis on the other hand shows no sign of degeneration. It is a small structure attached to the vas deferens. The internal lining of the right vas deferens is composed of normal ciliated columner epithelial cells. There is no sign of reduction in size.

I wish to express my sincere thanks to Professor D. R. Bhattacharya for his criticisms.

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Explanation of Figures

Fig. 1.—The abnormal testis of the lizard (*Hemidactylus flaviviridis* Ruppel). The right testis is very small as compared with the left.

Fig. 2.—Transverse section of the abnormal (right) testis.

Fig. 3.—A portion of the right testis under high power showing the degeneration of cells of the tubules. The intercellular spaces are filled with coagulated fluid.

Lettering

COG—Coagulated fluid in the centre of the tubules of the right testis.
CON—Connective tissue beneath the investing membrane.
DCT—Degenerate cells of the tubules of the right testis.
DT—Degenerate tubules of the right testis.
ITT—Intertubular tissue.
LT—Left testis.
LEPD—Left epididymis.
LVD—Left Vas deferens.
RT—Right testis.
REPD—Right epididymis.
RVD—Right Vas deferens.
TA—Tunica albuginia.

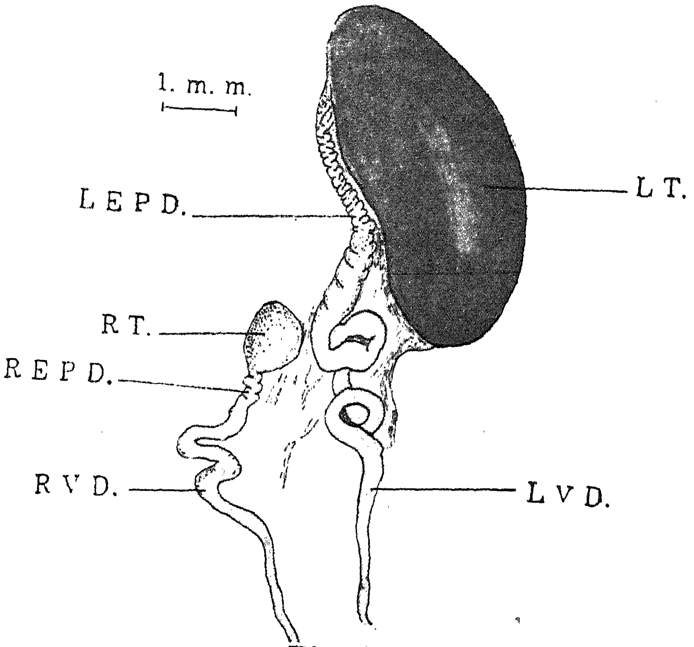

Fig. 1

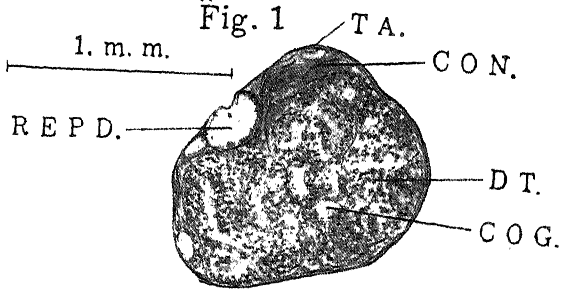

Fig. 2

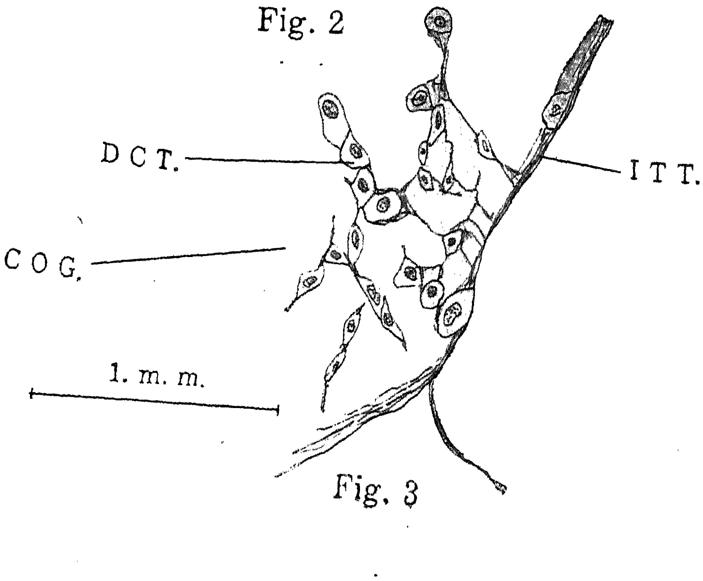

Fig. 3

ON A NEW SPECIES OF *CATATROPIS* ODHNER, 1905, FROM AN INDIAN FOWL—*GALLUS BANKIVA MURGHI*.

BY HAR DAYAL SRIVASTAVA

ZOOLOGY DEPARTMENT, UNIVERSITY OF ALLAHABAD

Communicated by Dr. H. R. Mehra

Received August 2, 1934

A large number of monostomes referable to *Catatropis* Odhner were obtained from the rectal caeca of the domestic fowl, *Gallus bankiva murghi*, which had died after a prolonged sickness. Besides these monostomes the bird was found on post-mortem examination to be heavily infected with *Amoebotaenia sphenoides* Railliet, 1892, and *Raillietina* (*Fuhrmannetta*) *echinobothrida* Megnin, 1800. The bird exhibited marked pathological symptoms such as pronounced emaciation, anaemia, dull plumage and poor health. There was little flesh on the breast and the legs were thin and dry. The intestines were badly damaged. In addition to an enormous enlargement of the rectal caeca intestinal inflammation and puss-formation were also noticed at several places.

Catatropis indicus N. Sp.

Notocotylidae is the most widely studied family amongst the monostomatous trematodes. It is in this family that we come across the earliest records of monostomes, such as *Catatropis verrucosa* Frolich, 1789, parasitic in the rectum of *Anas domestica*. Frolich, 1789, and Gmelin, 1790, classified these worms as *Fasciola* while Zeder in 1800 removed them to the genus *Monostoma*. In 1839 Diesing assigned these parasites to the genus *Notocotylus* of which *N. serialis* is the type species. Odhner in 1905 recognised this earliest known monostome to be distinct from Diesing's type species and removed it to a new genus *Catatropis*.

Upto this time half a dozen species have been recorded under this genus, namely:—*C. verrucosa* Frolich, *C. liara* Kossack, *C. charadrii* Scrjabin, *C. filamentis* Barker, *C. gallinulae* Johnston, and *C. orientalis* Harshe. In the following pages I give an account of one more species of the genus.

The parasite was found in large numbers in the rectal caeca of a domestic fowl. In the living condition they have a light brown colour and show little power of contraction and expansion. The flat, transparent body, 4·2–4·6* in length and 1·2 in maximum breadth which lies across the anterior end of the vitellaria, has smooth convex dorsal surface and an armed concave ventral surface bearing three longitudinal rows of unicellular glands which are non-protrusible. The glands in the median row are contiguous but those in the outer rows are distinct and vary from 10—12 in number in each row. The sides of the body are nearly parallel except at the ends which are bluntly rounded. The ventral surface is studded with minute backwardly directed spines, 0·008–0·01 in length and 0·002—0·003 in breadth at the base, extending between the region of the genital pore and the ovary. The ratio of the length to the maximum breadth of the body is as 4 : 1.

The excretory system is typical of the genus. The wide funnel-shaped bladder has its inner walls thrown into ridges forming the "Rippon" of Looss. From the anterior corners of the bladder are given off the cornua which lie laterally and give off throughout their course both internal and external branches. The excretory pore lies on the dorsal side about 0·2 in front of the posterior end.

The oral sucker is almost terminal with a spherical outline measuring 0·14—0·2 in diameter, and is followed by a 0·2—0·26 long oesophagus. Pharynx is absent. The intestinal caeca, of nearly equal length, end posteriorly about the level of the excretory pore. They are provided, throughout their course, with numerous minute diverticula and bend inwards in the region of the testes, lying between the latter and the median ovary and mehlis gland.

The deeply lobed testes, 0·75–0·99 in length and 0·2–0·3 in maximum breadth, are situated laterally near the posterior end close outside the intestinal caeca extending from the level of the mehlis gland to that of the excretory pore. The vesicula seminalis is enormously developed and lies outside the cirrus sac, extending in a characteristically coiled manner from the latter to the anterior limit of the vitellaria. The cirrus sac is

* All measurements are in m.m.

median and flask-shaped with a long neck, measuring 0·87—1·2 in length and 0·17—0·2 in maximum breadth across the bulb. The latter contains a cone-shaped pars prostatica of 0·35 × 0·09 size which is surrounded by well developed prostate gland cells. The genital pore lies in the median line close behind the oral sucker.

The irregularly lobed ovary, 0·26—0·35 in size, is situated in the intercaecal space posterior to the mehlis gland and in level with the middle thirds of the testes. From the anterior margin of the ovary arises a short and wide oviduct which enters the mehlis gland after giving off a short Laurer's canal. The mehlis gland, 0·23—0·26 in length and 0·17—0·26 in maximum breadth, is a somewhat triangular compact structure situated just in front of the ovary. The receptaculum seminis is absent.

The vitellaria are composed of small irregular follicles of 0·09—0·12 × 0·05—0·08 size which are arranged in a linear series, except at a few places, laterally outside the intestinal caeca. They begin from just behind the middle of the body and terminate a little beyond the anterior ends of the testes, extending over a length of 1·2—1·5. The transverse vitelline ducts arise from the posterior ends of the vitellaria and unite in the region of the mehlis gland to form an oval vitelline reservoir which lies to the left side in contact with the latter.

The first one or two coils of the uterus are filled with sperms forming the receptaculum seminis uterinum. The uterus, as in other members of the genus, is arranged in transverse coils in a characteristic manner in the intercaecal space between the gonads and the middle of the vesicula seminalis. Terminally it passes into a straight muscular metraterm of the same length as the cirrus sac. Eggs are small, thin-shelled, light brown in colour and are provided with long polar filament at each pole, measuring (excluding the filament) 0·017—0·02 × 0·008—0·01 in size.

C. indicus N. Sp. differs from all the known species of the genus in many features. In its relationship it stands nearest to *C. orientalis* which it resembles in the arrangement of ventral glands, size of sucker and oesophagus, position and shape of gonads and number of uterine coils. It differs, however, from the latter in shape and size of body, absence of ventral papillae, presence of body spines, size of gonads, length of cirrus sac, position of vitellaria and size of eggs. *C. indicus* can be distinguished from *C. gallinulae* by the position of testes, large size of vesicula seminalis, lobed character of ovary and size of eggs; from *C. filamentis* by the shape and size of body, size of gonads, character of

ovary and the caudad extent of vitellaria ; from *C. charadrii* by the shape and size of body, size and shape of gonads and size of vesicula seminalis ; from *C. verrucosa* by the diameter of sucker, size of vesicula seminalis and character of ovary. *C. indicus* differs markedly from all the known species of the genus in the position of the genital pore which lies far forward just behind the oral sucker.

This work was carried on under Dr. H. R. Mehra to whom my respectful gratitude is due for his kind help and suggestions. I am also grateful to Dr. D. R. Bhattacharya for laboratory facilities and to the Trustees of the Lady Tata Memorial Trust, Bombay, for the grant of a research scholarship in Helminthology.

Plate 1. Ventral View of Catatropis indicus N. Sp.

LETTERING

C.S.	Cirrus Sac.	O.S.	Oral Sucker.
G.P.	Genital Pore	P.G.	Prostate Glands.
I.C.	Intestinal Caecum.	P.P.	Pars Prostatica.
M.	Metraterm.	T.	Testis.
M.G.	Mehlis Gland.	U.	Uterus.
O.	Ovary.	V.	Vitellaria
Oes.	Oesophagus.	Y.R.	Yolk Reservoir.
O.D.	Oviduct.		

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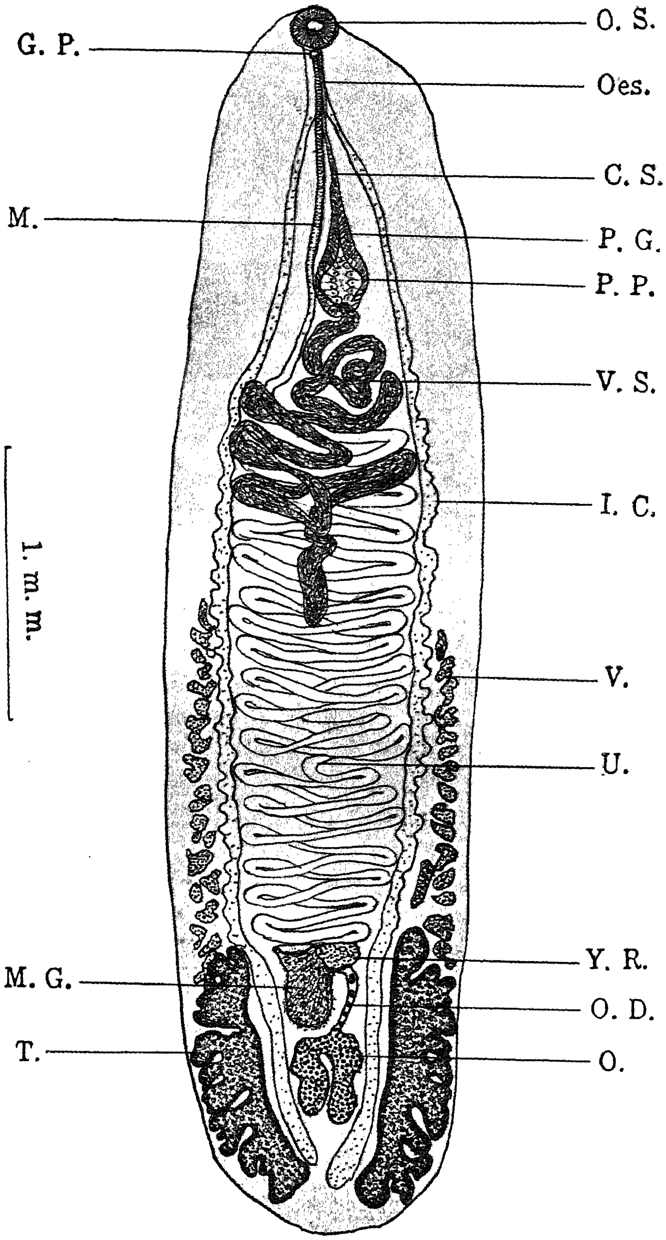

F. 11

A STUDY OF SOME ORGANIC REACTIONS AT LOW TEMPERATURES

By Cromwell Osborn Das and Sikhibhushan Dutt

Chemistry Department, Allahabad University

Received October 3, 1934

Organic chemistry since its inception has always been regarded as branch of science in which fairly high temperature has always played an important part in most of the reactions. We find that most of the processes employed therein, *e.g.*, distillation, fractioation, crystallisation, dry distillation, sublimation, digestion, extraction, etc., invariably require temperatures which are much above the ordinary. And so also do most of the reactions. Thus we find that Claisen's and Knoevenagel's reactions require temperatures ranging from 50°-60° to nearly 150°. Perkin's reaction requires heating at the boiling point of at least that of acetic anhydride (128°) for several hours in order to complete the reaction. In Friedel-Craft's reaction the constituents are heated at least to the boiling point of the dilutent, *i.e.*, carbondisulphide (46°) for several hours until the reaction is complete. Fittig's reaction often requires temperatures which nearly correspond to the boiling point of the halogenated benzene derivative (132° and upwards). Skraup's reaction requires temperature above 200° and Doebner-Miller's reaction requires prolonged heating at the boiling point of concentrated hydrochloric acid (110°-120°). Michael's reaction requires heating at least at the boiling point of the dilutent (ether—35°, benzene—80° and amyl ether—148°) for several hours. Reformatski's reaction is also identical with Grignard and Michael's reactions in this respect. Thorpe's reaction requires heating with 75-80% sulphuric acid (210°-230°). Reimer-Tiemann's reaction requires a temperature from 60° to 80° for several hours. Kolbe's reaction has an optimum temperature near about 200°, and Beckmann's transformation and Walden's inversion require temperatures near about the boiling point of ether or benzene. Hydrolysis of cane sugar is generally effected at 60° and hydrolysis of an ester with caustic potash at 70°-80°.

In short, most of the organic reactions, *e.g.*, hydrolysis, dehydration, condensation, etc., require temperatures above the ordinary. In very rare

instances we come across reactions which have been carried on at the melting point of ice. But we hardly find any example in which a reaction has been performed below 0°, *i.e.*, at a temperature of say $-6°$ to $-10°$. On account of the great paucity of data with regard to this point of view, the present investigation was undertaken in order to find out interesting types and examples of organic reactions which would go on well even at such low temperatures.

It has now been found in general that low temperature reactions studied in course of the present investigation are really somewhat slow as compared with the same carried on at higher temperatures. Of course this can be easily expected as heat is a great promoter of reactions. But there are some reactions which simply do not get on at low temperatures, *e.g.*, condensations of malonic acid with veratric aldehyde, *iso*valeraldehyde, *iso*butyraldehyde *p*-oxybenzaldehyde etc. In some cases it has been found that low temperature reactions are not quite analogous to the reactions carried on at higher temperatures, *e.g.*, *o*-nitrobenzaldehyde condenses with acetone at 40° in presence of dilute alkali with formation of the unsaturated ketone—*o*-nitrobenzylideneacetone; but the same reaction carried on at temperature of $-6°$ gave only indigo. Cane sugar which undergoes quantitative hydrolysis in 20 minutes at 60° by 5% hydrochloric acid, was altogether unaffected by the same reagent at $-6°$ in 20 hours. In the hydrolysis of esters of various description it was found that the velocity of hydrolysis was exceedingly reduced, *e.g.*, benzyl-cinnamate which undergoes complete hydrolysis at 80° by caustic potash (10%) in 12 minutes, was hydrolysed to the extent of only 9·8% in course of 12 days. Benzoin condensation which gave an yield of 90% after heating only for an hour at high temperature, gave only an yield of 35·5% at $-6°$ after 72 hours. Hydroquinone got oxidised to quinhydrone by ferric alum at $-6°$ to the extent of 73·3% after 24 hours, and quinone was reduced to hydroquinone by sulphurous acid to the extent of 58·3%. In some cases low temperatures have been found to be slightly more efficacious in conducting reactions than higher temperatures. Thus the formation of hydrobenzamide is 97·2% at $-6°$ to $-10°$, whereas the same reaction at 30° gave an yield of not more than 85%. The formation of phenoquinone is 61% at $-6°$ and 52% at 80°, of oxamide is 56·4% at $-6°$ and 47% at 60°.

The great difference between condensation products at low temperature and high temperature is that the crystalline structure of products formed in the former case is far more well marked and definite than in the latter. The amount of by-products formed is also much smaller at lower temperature, and in most of the cases no by-products are formed at all,

the compound obtained being exceedingly pure. The second difference lies in the great varieties of exhibitions of colour which low temperature reactions produce, before the final product is formed. The explanation for colour variations during the course of condensations can be found either in Vorländer's[1] hypothesis of formation of unstable intermediates or in Dewar's[2] hypothesis of different degrees of hydration of condensation products.

Lastly, the marked gradations in the yield with respect to the amount of condensing agent added is of great interest. In some cases it has also been possible to note that further addition of the condensing agent has no effect in the increase of the yield, after a certain percentage of yield is obtained.

The experimental portion of the investigation has been classified under the following four heads :

1. Condensation.
2. Oxidations, reductions, preparations.
3. Hydrolysis.
4. Transformations.

EXPERIMENTAL

The low temperature thermostat used in course of these experiments was a Frigidare refrigerator specially equipped and adjusted so as to give a temperature of -20° in the freezing coils and between -10° to -6° in the chamber. It was here that all the reactions described in this paper were carried out.

Condensation of cyclo-pentanone and ethyl-cyanacetate in presence of piperidine.—8·9 c.c. of an equimolecular mixture of cyclopentanone and ethyl-cyanacetate were taken in each one of 13 different flasks and they were kept in the Frigidare until they attained the inside temperature. Different quantities of well cooled piperidine were then added to the flasks and they were allowed to stand in the refrigerator for 24 hours. After that period dilute hydrochloric acid was added to each of the flasks in order to destroy the condensing agent and arrest the reaction. The reaction product from each of the flasks was submitted to steam distillation so as to remove the unreacted constituents. The residual condensation products were carefully recrystallised under identical conditions and the

yield noted in each case. The results are summarised in the table given below:

No. of flask.	Amount of piperidine added.	Weight of condensation product.	% yield.
1.	0·02 c. c.	2·6790 g.	14·00
2.	0·04 c. c.	4·8258 g.	37·80
3.	0·06 c. c.	5·4300 g.	44·50
4.	0·08 c. c.	6·1630 g.	60·83
5.	0·10 c. c.	6·5701 g.	65·34
6.	0·12 c. c.	6·9654 g.	69·70
7.	0·14 c. c.	7·1030 g.	71·24
8.	0·16 c. c.	7·2150 g.	72·40
9.	0·18 c. c.	7·2908 g.	73·30
10.	0·20 c. c.	7·3990 g.	74·40
11.	0·22 c. c.	7·4980 g.	76·22
12.	0·24 c. c.	7·5021 g.	76·32
13.	0·26 c. c.	7·5500 g.	76·40

Thus it is apparent from the above table that the yield of the condensation product increases with the increase of the amount of the condensing agent added until it comes to 0·22 c. c. After that further addition of piperidine has no effect in the increase of the yield of cyclopentylidene-ethyl-cyanacetate. The condensation product crystallises in colourless needles from alcohol melting at 52°.

Coumarin-carboxylic ester from salicylaldehyde and malonic ester in presence of piperidine.—This condensation was also effected in a manner similar to the above. The best yield was obtained when the proportion of piperidine to that of the other two constituents mixed together was in the ratio of 1 : 30, *i.e.*, 3·3%. The substance crystallises from acetone in colourless needles, melting at 52°. The optimum yield was 62·6%.

For the sake of abbreviation, the rest of the condensation work is summarised in tabular form. (The Duration of cooling was 24 hours in each case):

Condensation product.	Constituents	Condensing agent.	Optimum % of condensing agent.	Optimum yield. %	M. P. of condensation product.
Benzylidene-acetophenone	benzaldehyde & acetophenone	10% NaOH	6·8	70·4	57°
Benzylidene-ethyl-cyanacetate	benzaldehyde & ethyl-cyanacetate	6% NaOET	15·4	81·3	51°
Furylideneacetone	furfural & acetone	6% NaOET	19·0	40·2	91°

Condensation product.	Constituents	Condensing agent.	Optimum % of condensing agent.	Optimum yield %	M. P. of condensation product.
Benzylidene-acetone	benzaldehyde & acetone	ditto	17·8	51·2	42°
Piperonal acetone	piperonal & acetone	25 % NaOH	2·9	24·8	106°
Indigo	o-nitrobenzalde--hyde and acetone	ditto	2·4	38·4	...
Cinnamylidene-acetone	cinnamaldehyde & acetone	15 % NaOH	16·3	36·3	67°
Benzoin	benzaldehyde & KCN	10 % KCN	18·8	24·3	134°
Furylidene-ethylcyanacetate	furfural & ethyl-cyanacetate	10 % NaOH	13·7	70·8	93°
Benzylidene-aniline	benzaldehyde & aniline	6 % NaOET	16·0	83·7	54°

Oxidation

Quinhydrone from quinol.—2·5 grams of quinol dissolved in 25 c. c. of water and 3·62 grams of ferric alum dissolved in 10 c. c. of water were placed inside the refrigerator until the temperature was about −6°. They were then mixed together and allowed to stand at that temperature for 24 hours. The quinhydrone was then filtered off, washed and dried. M. P. 171°. Yield −173·3 %.

Heteroring Condensation

Hydrobenzamide from benzaldehyde and ammonia.—5 c.c., of freshly distilled benzaldehyde and 25 c.c. of strong ammonia were individually cooled and then mixed together and allowed to stand inside the refrigerator. After 24 hours the precipitated solid was filtered off, washed, dried and weighed. M. P. 105°. Yield—97·26%.

Polynuclear Condensation

Phenoquinone from phenol and quinone.—4·7 grams of pure phenol dissolved in 10 c.c. of petroleum ether and 2·7 grams of quinone also dissolved in 10 c.c. of petroleum ether were individually cooled and then mixed together. After 24 hours the precipitated crystalline phenoquinone was filtered off, washed with little petroleum ether, dried and weighed. M. P. 71·5°. Yield—61·0%.

Reduction

Quinol from quinone.—2 grams of quinone were treated with an icecold saturated aqueous solution of sulphur dioxide (30 c.c.) and the

mixture kept inside the refrigerator. After 24 hours the product was filtered from the unreacted quinone and from the filtrate the quinol was extracted with ether. After recrystallisation with the addition of a little animal charcoal the substance melted at 169°. Yield—58·32%.

Phenylhydrazine from diazobenzene chloride.—10 grams of aniline were diazotised as usual in dilute hydrochloric acid solution keeping the temperature below − 5°. 60 grams of stannous chloride dissolved in 30 c.c. of concentrated hydrochloric acid were also brought to the same temperature and then the two solutions were mixed together and the mixture allowed to stand inside the refrigerator. After 24 hours the crystalline precipitate was filtered off, washed with icecold concentrated hydrochloric acid and dried on a porous plate in the lime desiccator. The phenylhydrazine hydrochloride thus obtained was quite pure and was weighed. Yield—74·23%.

Hydrolysis of cane sugar.—5 grams of cane sugar dissolved in 100 c.c. of water and cooled to −6° were treated with 5 c.c. of concentrated hydrochloric acid (35·4%) also kept at the same temperature, and the mixed solutions allowed to stand inside the refrigerator. The amount of hydrolysis was ascertained from time to time by withdrawing a measured sample and after neutralisation, titrating it with standard Fehling's solution. The results are given below:

Time in hours	Percentage of hydrolysis.
24	2·65
48	23·46
72	35·62
96	39·84
120	58·83
144	66·81
216	76·93
288	80·00
384	90·90
480	96·86
720	99·86

From the above it can be said that cane sugar solution with 5 % strong hydrochloric acid requires 30 days for complete hydrolysis at −6°.

Hydrolysis of an ester, e.g., benzyl-cinnamate.—A 5% solution of benzyl-cinnamate in alcohol was treated with an equal volume of N/6·2 alcoholic caustic potash at −6° and the mixture kept in the refrigerator. From time to time a sample of the liquid was withdrawn and the amount of

hydrolysis produced in it was measured by titrating it against standard acid. The results are given below:

Time in hours	Percentage of hydrolysis.
24	1·6
48	3·6
72	7·6
120	9·0
172	9·81
240	9·98
afterwards	no further change.

Benzidine transformation.—2·5 grams of hydrazobenzene dissolved in 40 c.c. of 50% alcohol and 1·5 grams of stannous chloride dissolved in 3 c.c. of concentrated hydrochloric acid were individually cooled and then mixed together and kept inside the refrigerator. After 24 hours the precipitated crystalline solid was filtered off, washed with concentrated hydrochloric acid, and dried on a porous plate in the lime desiccator. The weight of the benzidine hydrochloride thus obtained was 2·40 grams which corresponds to an yield of 81·4%.

Beckmann's transformation—2·5 grams of acetophenone-oxime dissolved in 30 c.c. of anhydrous ether and 3·8 grams of phosphorus pentachloride were individually cooled and mixed together and the mixture allowed to stand in the refrigerator. After 48 hours the phosphorus pentachloride was destroyed by the addition of 30 gms. of ice and the ether removed by distillation. The insoluble product was filtered off, washed with dilute sodium hydroxide in order to remove any unchanged acetophenone oxime, and water, dried and weighed. The substance which was identified to be acetanilide weighed 0·336 gm., and melted at 115°. The yield correspond to 13·4 % of the theoretical.

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CHEMICAL EXAMINATION OF THE ROOTS OF *CITRULLUS COLOCYNTHIS* SCHRADER

By Radha Raman Agarwal and Sikhibhushan Dutt

Chemistry Department, Allahabad University

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Citrullus colocynthis, (N. O. Cucurbitaceæ) called Colocynth or bitter apple in English; *Indrayan* in Hindi and Bengali; *Hanzal* in Persian; and *Indravaruni* or *Vishala* in Sanskrit, is a plant used in medicine for a very long time. It is grown wild in waste tracts of North-West, Central and South Indies, and Beluchistan. The fruit is globular of the size of an orange when ripe. The root is fibrous, tough stingy of an yellowish white colour. All parts of the plant are very bitter.

The fruit of this plant has been the subject of many investigations. Walz[1] claimed to have isolated a glucoside called colocynthin in 1858, which was evidently an amorphous mass. Later on in 1883 Hencke[2] examined the fruits and failed to obtain the glucoside isolated by Walz. Johannson[3], two years later, obtained the glucoside colocynthin and showed that this on hydrolysis yields the aglucone colocynthein along with elaterin and bryonin. Nayor and Chappel[4], working on the Indian variety, succeeded in confirming the results of Walz[1] by a modification of the method adopted by Hencke[2], and isolated the so-called glucoside in a crystalline form. They also stated that their product on hydrolysis yielded amongst other substances, colocynthein and elaterin, thus confirming the results of Johannson[3]. More recently Power and Moore[5] thoroughly examined the pulp of the fruits of Colocynth. In the course of their elaborate investigation they isolated the following substances: a di-hydroxy alcohol citrullol, $C_{22}H_{36}O_2(OH)_2$, m. p. 285-290°; an amorphous alkaloid, α-elaterin, m. p. 230°; hentriacontane, $C_{31}H_{64}$, m. p. 68°, and a phytesterol, $C_{27}H_{46}O$, m. p. 160-162°.

In spite of so much literature on the chemical examination of the fruits of Colocynth, the roots of the plant remain untouched. The only reference found is a note by Dymock[6], in which he mentions that he "examined the roots dried at 50°C and reduced to powder; the powder contains a large amount of starch and woody fibre. Ether extracted only

0·14% of a bitter oily matter. Dilute alcohol extracted 12·62% of a soft yellow non-crystalline mass dried at 100°. By the action of cold water on the extract 0·88% of insoluble soft yellow residue was left, which was not bitter and had acidic reactions—a fatty acid." In view of the fact that the roots are also put to a great medicinal use in India and elsewhere, it was deemed proper by the present authors to put it to a thorough chemical examination.

As regards the medicinal properties of the roots, "Sanskrit writers describe it as a useful cathartic in jaundice, ascites, enlargement of the abdominal viscera, urinary diseases and rheumatism, etc. Mohammedan writers consider the plant to be a very drastic purgative removing phlegm from all parts of the system and direct the fruit, leaves and roots to be used. A paste of the root is applied to the enlarged abdomen of children" (Dymock[6]).

The present investigation has shown that the roots contain like the fruits a considerable quantity of α-elaterin (0·2%), an amorphous saponin (1·2%), hentriacontane, inorganic materials and reducing sugars. The α-elaterin is not present in so huge quantity as that reported from the pulp of the fruit which was about one per cent. The preliminary examination showed the presence of alkaloids in the root, but all attempts to isolate them failed.

The constitution of α-elaterin has not yet been definitely established. F. Von Hemmelmayer[7] prepared from it a di-acetyl derivative which was not crystalline. We have, however, been successful in obtaining the diacetyl derivative in a crystalline form by heating together, α-elaterin and acetic anhydride with fused sodium acetate for 18 hours over a sand bath and crystallising the product from acetic acid.

Experimental

The roots were obtained from the local market and were finely crushed in an iron mortar. When burnt completely in a porcelain dish there was left 10·01% of a dirty white inorganic ash. The ash contained 12% of water soluble and 88% of water insoluble inorganic matter. The following elements and radicals were detected in the ash: chloride, sulphate, carbonate, potassium, magnesium, aluminium (traces), iron, calcium and silica.

In order to ascertain the general characteristics of the soluble portion of the roots, samples of finely powdered material were exhaustively extracted in a Soxhlet extraction apparatus using various solvents.

The following statement contains the amount of extract dried at 100° obtained:

1. *Petroleum Ether Extract.*—1·01%. The extract was oily and contained a lot of chlorophyll and waxy matters.

2. *Benzene Extract.*—3·30%. A green extract was obtained having properties similar to the petroleum extract obtained above.

3. *Acetone Extract.*—8·78%. Brownish red extract, giving characteristic smell. Gave reactions for carbohydrates, glucosides, alkaloids and had properties similar to the alcoholic extract.

4. *Alcoholic Extract.*—12·9%. Brownish yellow extract giving the characteristic smell of the plant. It contained some crystalline matter suspended in it. It gave a precipitate with lead acetate, and silver nitrate, reduced Fehling's solution. Gave a green colouration with ferric chloride, a precipitate with phosphotungstic acid and Mayer's reagent.

5. *Aqueous Extract.*—4·1%. A brown coloured extract. It reduced Fehling's solution easily, showing the presence of large amount of reducing sugars; also gave reactions for saponins. Formed a violet colouration with α-naphthel in chloroform solution and concentrated sulphuric acid. A precipitate was formed with lead acetate.

A preliminary examination for alkaloids was made with 200 grams of the powdered stuff. The alcoholic extract was diluted with water and treated with a little hydrochloric acid. It was then tested with various alkaloid reagents whereby the following precipitates or colouration were observed, showing definitely the presence of an alkaloidal body in the roots.

Alkaloidal reagents.	*Remarks.*
Fröhdes reagent.	A brownish colouration.
$KI + I_2$.	A reddish brown precipitate.
Mendelin's reagent.	A brownish colour with some precipitate.
Phosphotungstic acid.	A green precipitate.
Phospho-molybdic acid.	A white precipitate.
Mayer's reagent.	A dirty white precipitate.
Con. $H_2SO_4 + K_2Cr_2O_7$.	No change.
Con. $H_2SO_4 + HNO_3$.	No change.
Dragendroff's reagent.	A deep brown precipitate.
Sodium bicarbonate.	No change.
Picric acid.	A yellow precipitate.

For a complete analysis 2 kilograms of the powdered roots were exhaustively extracted with boiling alcohol in a big extraction flask of five litre capacity in two lots of 1 kilogram each. Seven extractions were necessary to remove all the soluble portions. The combined alcoholic extract on partial removal of the solvent yielded a crystalline stuff which was filtered, washed and dried when it melted at 227°C. It amounted to 0·6 grams. It was recrystallised from boiling alcohol when it was obtained as a white crystalline powder melting sharp at 229—230°C. It was later on identified to be α-elaterin.

Isolation of a hydrocarbon.—The alcoholic extract on the removal of the crystalline stuff, was evaporated to dryness on a water-bath, when it was obtained as a dark brown highly hygroscopic mass. It was then refluxed with petroleum ether, in order to remove chlorophyll and other oily constituents. The petroleum ether extract, which was light green in colour, slowly deposited a small amount of a white crystalline sediment. It was then filtered and the dirty white sediment recrystallised from hot petroleum ether whereby it was obtained in white flakes melting at 66-67°C. It was most probably the hydrocarbon hentriacontane $C_{31}H_{64}$ isolated by Power and Moore[5] from the fruits. However, the quantity at our disposal was too small for any further investigation.

Isolation of α-elaterin.—The resinous mass left after treating the alcoholic extract with petroleum ether was extracted with ethyl acetate till ethyl acetate ceased to dissolve anything. The combined ethyl acetate extract on concentration deposited a white powder, which was filtered, washed and dried in vacuum. It was recrystallised from hot boiling alcohol whereby a small amount (3 grams) of micro-crystalline powder was obtained melting sharp at 229-230°C. It was very little soluble in alcohol, acetone, and ethyl acetate and practically insoluble in petroleum ether, benzene and water. It gave a yellow colouration with concentrated sulphuric acid, which on heating deposited some amorphous mass. It dissolved on boiling in caustic potash with a deep red colouration. It reduced Tollen's reagent readily and gave a red colouration with alkaline potassium nitroprusside. From all its properties, colour reactions and elementary analysis, the stuff was shown to be α-elaterin (Found: C, 69·0; H, 7·5; $C_{28}H_{38}O_7$ requires C, 69·1; H, 7·8%).

This substance was lævo-rotatory and gave $[\alpha]_D^{28} = -62{\cdot}3°$ in chloroform solution in a 1 dm. tube. In order to confirm the identity of this substance with α-elaterin, the acetyl derivative was prepared in the usual way.

Di-acetyl α-elaterin.—0·5 grams of the α-elaterin was heated in a round-bottomed flask with excess of acetic anhydride and fused sodium

acetate. The mixture on cooling was poured in excess of water, whereby a brown amorphous product was obtained. It was crystallised from acetic acid when rhombic plates were obtained melting at 122-123° (c. f. F. Von. Hemmelmayer,[7] [Found: C, 67·10; H, 7·80; $C_{32}H_{42}O_9$ requires C, 67·43; H, 7.4%].

The crystalline stuff obtained from the alcoholic extract in the beginning which melted at 229°C. was also suspected to be α-elaterin. In order to establish this a little of the former was mixed with a little of the latter and the melting point observed was not depressed. The two products were therefore mixed.

The ethyl acetate extract, from which α-elaterin was separated, was evaporated to dryness, when a resinous material was obtained. This was dissolved in hot acetone. At this stage a white sediment (0·2 gram) remained undissolved which was identified as α-elaterin. Nothing definite could be separated from this acetone extract.

Isolation of Saponin.—The dried alcoholic extract which remained undissolved in ethyl acetate as described above, was dissolved in water with constant stirring and treated with a solution of lead acetate when no precipitate was formed. On treating it with basic lead acetate, however, a flocculant yellow precipitate was obtained which was filtered and washed thoroughly. This lead salt was suspended in water and decomposed as usual with sulphuretted hydrogen. The precipitated lead sulphide was filtered and washed. The filtrate obtained after the removal of the lead sulphide was concentrated under a very high vacuum, when a dark brown stuff was obtained on complete removal of water. It was dried in vacunm over calcium chloride. This resinous mass consisted mostly of saponins since on shaking it with water a large amount of frothing took place. A red colouration was obtained in cold on the addition of concentrated sulphuric acid. A Turnbulls blue colouration was developed on the addition of potassium ferricyanide containing a little ferric chloride. All these reactions clearly showed the presence of a large amount of saponins. An attempt was made to isolate it in a state of purity by precipitation with an alcoholic solution of cholesterol as cholesterolide but could not be met with success.

The filtrate obtained after the separation of the basic lead salt was treated with hydrogen sulphide to remove the excess of lead and filtered. It reduced Fehling's solution easily showing the presence of a large amount of reducing sugars.

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SUMMARY

From the roots of *Citrullus colocynthis*, a hydrocarbon, hentriacontane $C_{31}H_{64}$, α-elaterin $C_{28}H_{38}O_7$ and amorphous saponin have been isolated.

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